SRI RAAJA RAAJAN COLLEGE OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

Minutes of Subject allocating meeting

I **P.PRADEEP CASTRO hereby** submit the minutes of department meeting held forthe subject allocating on 19.05.2020 at 10.00 am based on the annexure I & annexure II.

S.NO	NAME OF THE STAFF	NAME OF THE SUBJECT	CLASS	NO OF HRS	STAFF SIGN
1	**P.Pradeep castro**	ME 8594-DYNAMICS OF MACHINES	III	5	

P. Pradeep Castro

HOD SIGN

PRINCIPAL
Sri Sri Raaja Raajan College of Engg & Tech
Amaravathipudur, Karaikudi - 630 301
Sivagangai, Tamil Nadu

CIRCULAR

DATE: 19.05.2020

Intimation of course allotment for faculties in the Department of Mechanical Engineering during ODD SEMESTER for B.E-MECHANICAL ENGINEERING.

S.NO	NAME OF THE FACULTY	TITLE OF COURSE	COURSE CODE	YEAR/SEM
1	(PRINCIPAL)	Dynamics of machines Mechatronics	ME 8594 ME 8791	III/V IV/VII
2	P.PRADEEP CASTRO ASST-PROFESSOR & MECH	Dynamics of machines Mechatronics	ME 8594 ME 8791	III/V IV/VII

Note: Faculties are asked to follow the syllabus issued by Anna University, Chennai.

Copy to

1. The HOD
2. All the faculties of Mechanical Engineering Dept.
3. File copy

SRI RAJA RAJAN COLLEGE OF ENGINEERING AND TECHNOLOGY KARAIKUDI

PRINCIPAL
Sri Raja Rajan College of Engg. & Tech
Amaravathipudur, Karaikudi-630 301
Sivagangai Dist. Tamil Nadu

ME8594 **DYNAMICS OF MACHINES**

L	T	P	C
4	0	0	4

OBJECTIVES:

- To understand the force-motion relationship in components subjected to external forces and analysis of standard mechanisms.
- To understand the undesirable effects of unbalances resulting from prescribed motions in mechanism.
- To understand the effect of Dynamics of undesirable vibrations.
- To understand the principles in mechanisms used for speed control and stability control

UNIT I FORCE ANALYSIS **12**

Dynamic force analysis – Inertia force and Inertia torque– D Alembert's principle –Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod– Bearing loads – Crank shaft torque – Turning moment diagrams –Fly Wheels – Flywheels of punching presses- Dynamics of Cam- follower mechanism.

UNIT II BALANCING **12**

Static and dynamic balancing – Balancing of rotating masses – Balancing a single cylinder engine – Balancing of Multi-cylinder inline, V-engines – Partial balancing in engines – Balancing of linkages – Balancing machines-Field balancing of discs and rotors.

UNIT III FREE VIBRATION **12**

Basic features of vibratory systems – Degrees of freedom – single degree of freedom – Free vibration– Equations of motion – Natural frequency – Types of Damping – Damped vibration– Torsional vibration of shaft – Critical speeds of shafts – Torsional vibration – Two and three rotor torsional systems.

UNIT IV FORCED VIBRATION **12**

Response of one degree freedom systems to periodic forcing – Harmonic disturbances – Disturbance caused by unbalance – Support motion –transmissibility – Vibration isolation vibration measurement.

UNIT V MECHANISM FOR CONTROL **12**

Governors – Types – Centrifugal governors – Gravity controlled and spring controlled centrifugal governors – Characteristics – Effect of friction – Controlling force curves. Gyroscopes –Gyroscopic forces and torques – Gyroscopic stabilization – Gyroscopic effects in Automobiles, ships and airplanes.

TOTAL : 60 PERIODS

OUTCOMES:

Upon the completion of this course the students will be able to

CO1 Calculate static and dynamic forces of mechanisms.
CO2 Calculate the balancing masses and their locations of reciprocating and rotating masses.
CO3 Compute the frequency of free vibration.
CO4 Compute the frequency of forced vibration and damping coefficient.
CO5 Calculate the speed and lift of the governor and estimate the gyroscopic effect on automobiles, ships and airplanes.

TEXT BOOKS:

1. F. B. Sayyad, "Dynamics of Machinery", McMillan Publishers India Ltd., Tech-Max Educational resources, 2011.
2. Rattan, S.S, "Theory of Machines", 4th Edition, Tata McGraw-Hill, 2014.
3. Uicker, J.J., Pennock G.R and Shigley, J.E., "Theory of Machines and Mechanisms", 4th Edition, Oxford University Press, 2014.

REFERENCES:

1. Cleghorn. W. L, "Mechanisms of Machines", Oxford University Press, 2014
2. Ghosh. A and Mallick, A.K., "Theory of Mechanisms and Machines", 3rd Edition Affiliated East-West Pvt. Ltd., New Delhi, 2006.
3. Khurmi, R.S.,"Theory of Machines", 14th Edition, S Chand Publications, 2005.
4. Rao.J.S. and Dukkipati.R.V. "Mechanisms and Machine Theory", Wiley-Eastern Ltd., New Delhi, 1992.
5. Robert L. Norton, "Kinematics and Dynamics of Machinery", Tata McGraw-Hill, 2009.
6. V.Ramamurthi, "Mechanics of Machines", Narosa Publishing House, 2002.

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
ACADEMIC YEAR: 2022 - 2023 ODD SEM
TIMETABLE

III YEAR/ V TH SEM **BATCH: 2020- 2024**

CLASS COORDINATOR: P. PRADEEP CASTRO

TIME/ DAY	9:30AM TO 10:20AM	10:20AM T0 11:10AM	BREAK	11:20AM TO 12:10PM	12:10PM TO 1:00PM	LUNCH	1:35PM TO 2:20PM	2:20PM TO 3:05PM	BREAK	3:15PM TO 4:00PM
MON	DOM	DOM		DME	DME		RES	KOM LAB		MM LAB
TUE	DME	TE-2		DOM	DOM		TE-2	MM		RES
WED	RES	MM		TE-2	TE-2		DME	MM LAB		TE LAB
THU	MM	TE-2		DOM	RES		LIB	MM		DOM
FRI	TE-2	RES		DME	DME		MM	TE LAB		KOM LAB

S.NO	SUB CODE	SUBJECT NAME	NAME OF STAFF	NO. OF HRS
THEORY				
1	ME8595	Thermal Engineering- II (TE-2)	Mr. S. SUNDARAVIGNESH	6
2	ME8593	Design of Machine Elements (DME)	Mr. D. SIVARAMAN	6
3	ME8501	Metrology and Measurements (MM)	Mr. S. PREM	5
4	ME8594	Dynamics of Machines (DOM)	Mr.PRADEEP CASTRO	6
5	ORO551	Open Elective I: Renewable Energy Sources (RES)	Mr.P. SARAN KUMAR	5
PRACTICALS				
1	ME8511	Kinematics and Dynamics Laboratory	Mr. S. PREM	4
2	ME8512	Thermal Engineering Laboratory	Mr.PRADEEP CASTRO	4
3	ME8513	Metrology and Measurements Laboratory	Mr. S. SUNDARAVIGNESH	4

TIMETABLE CO-ORDINATOR HOD DEAN ACADEMIC CO-ORDINATOR PRINCIPAL

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
ACADEMIC YEAR: 2022 - 2023 ODD SEM
TIMETABLE

P. PRADEEP CASTRO

TIME/ DAY	9:30AM TO 10:20AM	10:20AM T0 11:10AM	BREAK	11:20AM TO 12:10PM	12:10PM TO 1:00PM	LUNCH	1:35PM TO 2:20PM	2:20PM TO 3:05PM	BREAK	3:15PM TO 4:00PM
MON	DOM	DOM			MTS		EMM			EMM
TUE				DOM	DOM			MTS LAB		MTS LAB
WED		EMM			MTS			TE LAB		TE LAB
THU	EMM	MTS		DOM			EMM			DOM
FRI		MTS			MTS			TE LAB		MTS

S.NO	SUB CODE	SUBJECT NAME	YEAR/ SEM	NO. OF HRS
THEORY				
1	ME8791	Mechatronics (MTS)	IV YR / VII SEM	5
2	ME8594	Dynamics of Machines (DOM)	III YR / V SEM	6
3		Engineering Materials &Metallurgy (EMM)	II YR / III SEM	5
PRACTICALS				
2	ME8781	Mechatronics Laboratory (MTS LAB)	IV YR / VII SEM	2
3	ME8512	Thermal Engineering Laboratory (TE LAB)	III YR / V SEM	4

TIMETABLE CO-ORDINATOR HOD DEAN ACADEMIC CO-ORDINATOR PRINCIPAL

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING & TECHNOLOGY
AMARAVATHIPUDUR, KARAIKUDI-630 301
DEPARTMENT OF MECHANICAL ENGINEERING
STUDENT NAME LIST

III YEAR/ V SEM **BATCH 2020 -2024**

S.NO.	REGISTER NUMBER	NAME
1	912520114001	N.ASWIN
2	912520114002	AV.BHARANITHARAN
3	912520114003	C.DHARMARAJ
4	912520114004	A.GANESH KUMAR
5	912520114005	B.GOWTHAM
6	912520114006	V.GUNA
7	912520114008	S.HARIHARAN
8	912520114009	R.KARTHIKEYAN
9	912520114011	R.KAVINKUMAR
10	912520114012	K.KAVIYARASAN
11	912520114013	A.MANIKANDAN
12	912520114014	S.MUTHUSETHUPATHY
13	912520114015	PANDIKUMAR
14	912520114016	N.PRAKASH
15	912520114017	S.PRAVEEN KUMAR
16	912520114018	R.RAGUL RAJ
17	912520114019	I.SAKTHIVEL
18	912520114020	R.SANTHOSH KUMAR
19	912520114021	N.SARAN
20	912520114022	G.SELVA KUMAR
21	912520114024	B.SETHUPATHY
22	912520114025	P.SRIRAMAN
23	912520114026	SUKUMARAN
24	912520114027	R.SURESH
25	912520114028	P.VASANTH
26	912520114029	V.VIGNESH
27	912520114030	R.VISWA
28	912520114301	ARUNKUMAR S
29	912520114302	ARUN SELVAKUMAR C
30	912520114303	BHARATHI M

31	912520114304	GOWTHAM R
32	912520114305	HARIHARAN M (5/6/2000)
33	912520114306	HARIHARAN M
34	912520114307	HARIHARAN V
35	912520114308	KAILAI NATHAN M
36	912520114309	KAMALESH S
37	912520114310	KIRUBAKARAN J
38	912520114311	LOGESHWARAN R
39	912520114312	MATHAN M
40	912520114313	MAYARAJA K
41	912520114314	MOHAMED SATHIK A
42	912520114315	MOHAMMED AASHIK J
43	912520114316	MUTHU MANICKAM P
44	912520114317	NATHER KADAR BATCHA M
45	912520114318	NAVEEN SHANKAR K
46	912520114319	NESAPRAKASH M
47	912520114320	OYYAPPAN M
48	912520114321	PARAMASIVAM PL
49	912520114323	PARTHIBAN M
50	912520114324	RAMKUMAR P
51	912520114325	RISHI R
52	912520114326	SABARISH R
53	912520114327	SANTHOSH R
54	912520114328	SATHYAN A
55	912520114329	SELVAMURUGAN T
56	912520114330	SIVAKUMAR R
57	912520114331	SOMASUNDARA RAJAN S
58	912520114332	VENKATESH M
59	912520114333	VIGNESH S
60	912520114334	VIGNESH RAJ R
61	912520114335	VIGNESHWARAN U
62	912520114336	VISHNU R

for

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING & TECHNOLOGY, AMARAVATHIPUDUR, KARAIKUDI

DEPARTMENT OF MECHANICAL ENGINEERING

Date:

Name of the Staff : P. PRADEEP CASTRO

Programme : B.E.

Course Code &Title : ME8594 DYNAMICS OF MACHINES (R2017)

Year / Semester : III YEAR/ V SEM

COURSE PLAN

1. VISION& MISSION OF THE DEPARTMENT

VISION:-

To provide broad-based education and training in Mechanical Engineering and its applications to enable the graduates meet the demands in a rapidly changing needs in industry, academia and society.

MISSION:-

- Impart high quality technical education and training that encompasses both theory and practice with an emphasis on human and social values.
- To equip the students to face future technology and global changes.
- To create, explore, and develop innovations in mechanical engineering research.

2. PROGRAM OUTCOMES

1. An ability to apply knowledge of mathematics and engineering sciences to develop mathematical models for industrial problems.
2. An ability to identify, formulates, and solve complex engineering problems. with high degree of competence.
3. An ability to design and conduct experiments, as well as to analyze and interpret data obtained through those experiments.
4. An ability to design mechanical systems, component, or a process to meet desired needs within the realistic constraints such as environmental, social, political and economic sustainability.
5. An ability to use modern tools, software and equipment to analyze multidisciplinary problems.
6. An ability to demonstrate on professional and ethical responsibilities.

7. An ability to communicate, write reports and express research findings in a scientific community.

8. An ability to adapt quickly to the global changes and contemporary practices.

9. An ability to engage in life-long learning.

Program Specific Outcomes:-

PSO1: To analyze, design and develop computing solutions by applying foundational concepts Mechanical Engineering.
PSO2: To apply engineering principles and practices for developing quality products and advanc manufacturing facilities
PSO 3: To adapt to emerging trends to innovate ideas and solutions to existing / novel problems

3. PRE-REQUISITE

To understand the force-motion relationship in components subjected to external forces a analysis of standard mechanisms.

4. OBJECTIVES

- To understand the undesirable effects of unbalances resulting from prescribed motions mechanism.
- To understand the effect of Dynamics of undesirable vibrations.
- To understand the force-motion relationship in components subjected to external forces analysis of standard mechanisms
- To understand the principles in mechanisms used for speed control and stability control.

5. COURSE OUTCOMES

Upon the completion of this course the students will be able to

CO1: Calculate static and dynamic forces of mechanisms.

CO2: Calculate the balancing masses and their locations of reciprocating and rotating masses.

CO3: Compute the frequency of free vibration.

CO4: Compute the frequency of forced vibration and damping coefficient.

CO5: Calculate the speed and lift of the governor and estimate the gyroscopic effect on automo ships and airplanes

6. MAPPING OF PO'S & CO'S

Course Outcomes	Program Outcomes (POs)								
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9
CO 1	✓	✓		✓			✓		
CO 2	✓	✓	✓		✓	✓			
CO 3	✓	✓		✓			✓	✓	✓
CO 4	✓		✓		✓	✓			
CO 5	✓	✓	✓		✓	✓	✓	✓	

7. CURRICULUM GAPS

- Dynamic analysis of Single cylinder two stroke & four stroke engines.

8. SYLLABUS

UNIT I FORCE ANALYSIS

Dynamic force analysis – Inertia force and Inertia torque– D Alembert's principle –Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod– Bearing loads – Crank shaft torque – Turning moment diagrams –Fly Wheels – Flywheels of punching presses- Dynamics of Cam- follower mechanism.

UNIT II BALANCING

Static and dynamic balancing – Balancing of rotating masses – Balancing a single cylinder engine – Balancing of Multi-cylinder inline, V-engines – Partial balancing in engines – Balancing of linkages – Balancing machines-Field balancing of discs and rotors.

UNIT III FREE VIBRATION

Basic features of vibratory systems – Degrees of freedom – single degree of freedom – Free vibration– Equations of motion – Natural frequency – Types of Damping – Damped vibration– Torsional vibration of shaft – Critical speeds of shafts – Torsional vibration – Two and three rotor torsional systems.

UNIT IV FORCED VIBRATION

Response of one degree freedom systems to periodic forcing – Harmonic disturbances – Disturbance caused by unbalance – Support motion –transmissibility – Vibration isolation vibration measurement.

UNIT V MECHANISM FOR CONTROL

Governors – Types – Centrifugal governors – Gravity controlled and spring controlled centrifugal governors – Characteristics – Effect of friction – Controlling force curves. Gyroscopes –Gyroscopic forces and torques – Gyroscopic stabilization – Gyroscopic effects in Automobiles, ships and airplanes.

List of Text Books by AU:

T1. F. B. Sayyad, "Dynamics of Machinery", McMillan Publishers India Ltd., Tech-Max Educational resources, 2011.

T2. Rattan, S.S, "Theory of Machines", 4 th Edition, Tata McGraw-Hill, 2014

T3. Uicker, J.J., Pennock G.R and Shigley, J.E., "Theory of Machines and Mechanisms", 4 th Edition, Oxford University Press, 2014

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R2. Ghosh. A and Mallick, A.K., "Theory of Mechanisms and Machines", 3 rd Edition Affiliated East-West Pvt. Ltd., New Delhi, 2006.

R3. Khurmi, R.S.,"Theory of Machines", 14th Edition, S Chand Publications, 2005.

R4. Rao.J.S. and Dukkipati.R.V. "Mechanisms and Machine Theory", Wiley-Eastern Ltd., New Delhi, 1992.

R5. Robert L. Norton, "Kinematics and Dynamics of Machinery", Tata McGraw-Hill, 2009.

R6. V.Ramamurthi, "Mechanics of Machines", Narosa Publishing House, 2002.

9. LESSON PLAN

Lecture No.	Topic(s) to be covered	Text / Ref. Book	Page No.	Teaching Aid	Learning Mode	No. of Hrs	Course Outcomes
	UNIT I FORCE ANALYSIS						
1	Dynamic force analysis – Inertia force and Inertia torque	T1	3-10	BB	Interactive	2	CO 1
2	D Alembert's principle	T1	11-12	BB	Collaborative	1	CO 1
3	Dynamic Analysis in reciprocating engines	T1	24-40	BB	Collaborative	2	CO 1
4	Gas forces – Inertia effect of connecting rod	T1	111-120	BB	Interactive	2	CO 1
5	Bearing loads – Crank shaft torque	T1	90-96	BB	Collaborative	1	CO 1
6	Turning moment diagrams –Fly Wheels	T1	63-73	BB	Interactive	2	CO 1
7	Flywheels of punching presses	T1	80-87	BB	Interactive	2	CO 1
No. of hours planned .....No. of hours taken.....Deviation..... **Reason.....**							
	UNIT II BALANCING						
8	Static and dynamic balancing	T1	178	BB	Interactive	1	CO 2
9	Balancing of rotating masses	T1	178-183	BB	Interactive	2	CO 2

10	Balancing a single cylinder engine	T1	183-189	BB	Collaborative	2	CO 2
11	Balancing of Multi-cylinder inline, V-engines	T1	183-189	BB	Interactive	2	CO 2
12	Balancing a single cylinder engine	T1	189-196	BB	Interactive	1	CO 2
13	Partial balancing in engines – Balancing of linkages	T1	189-196	BB	Interactive	2	CO 2
14	Balancing machines	T1	196-222	BB	Interactive	1	CO 2
15	Field balancing of discs and rotors	T1	225-229	BB	Interactive	1	CO 2

No. of hours planned .....No. of hours taken.....Deviation.....
Reason.....

UNIT III FREE VIBRATION

16	Basic features of vibratory systems – Degrees of freedom – single degree of freedom	T1	242-244	BB	Interactive	1	CO 3
17	Free vibration– Equations of motion - Natural frequency	T1	248-251	BB	Interactive	2	CO 3
18	Types of Damping – Damped vibration	T1	251-259	BB	Interactive	2	CO 3
19	Free vibration– Equations of motion - Natural frequency	T1	259-272	BB	Interactive	2	CO 3
20	Types of Damping – Damped vibration	T1	272-286	BB	Interactive	2	CO 3
21	Torsional vibration of shaft – Critical speeds of shafts	T1	286-303	BB	Interactive	2	CO 3
22	Torsional vibration – Two and three rotor torsional systems.	T1	303-316	BB	Collaborative	1	CO 3

No. of hours planned .....No. of hours taken.....Deviation.....
Reason.....

UNIT IV FORCED VIBRATION

23	Response of one degree freedom systems to periodic forcing	T1	374-378, 418-424	BB	Interactive	3	CO 4
24	Harmonic disturbances	T1	378-383	BB	Interactive	2	CO 4
25	Disturbance caused by unbalance	T1	383-418	BB	Interactive	3	CO 4

[illegible]	Support motion transmissibility	[illegible]	[illegible]	[illegible]	Collaborative	[illegible]	[illegible]
[illegible]	Vibration isolation, vibration measurement.	[illegible]	[illegible]	[illegible]	Collaborative	[illegible]	[illegible]

No. of hours planned .... No. of hours taken .... Deviation ....
Reason ....

UNIT V MECHANISM FOR CONTROL

[illegible]	Governors - Types	[illegible]	[illegible]	[illegible]	Collaborative	[illegible]	[illegible]
[illegible]	Centrifugal governors - Gravity controlled and spring controlled centrifugal governors - Characteristics	[illegible]	[illegible]	[illegible]	Interactive	[illegible]	[illegible]
[illegible]	Effect of friction - Controlling force curves	[illegible]	[illegible]	[illegible]	Collaborative	[illegible]	[illegible]
[illegible]	Gyroscopes - Gyroscopic force and couple	[illegible]	[illegible]	[illegible]	Interactive	[illegible]	[illegible]
[illegible]	Gyroscopic stabilization	[illegible]	[illegible]	[illegible]	Interactive	[illegible]	[illegible]
[illegible]	Gyroscopic effects in Automobiles	[illegible]	[illegible]	[illegible]	Interactive	[illegible]	[illegible]

No. of hours planned .... No. of hours taken .... Deviation ....
Reason ....

III. WEB REFERENCES

Unit	Topic	Web Link
I	D'Alembert's principle	https://collegedunia.com/[illegible]
II	[illegible]	https://www.[illegible]
III	Basic features of vibratory systems	http://[illegible].pdf
IV	Vibration isolation	https://[illegible]
V	Gyroscopic effects in Automobiles	https://www.[illegible]

Video Resources

S.No.	Video Link
V1	**D Alembert's principle :** https://www.youtube.com/watch?v=PlbgIGH4kmE
V2	**Balancing of Multi-cylinder inline engines :** https://www.youtube.com/watch?v=kyM8i4o_7MY
V3	**Degrees of freedom :** https://www.youtube.com/watch?v=rATNoxKg1yA
V4	**Harmonic disturbances :** https://www.youtube.com/watch?v=vkh8Jfmm3hk
V5	**Gyroscopes :** https://www.youtube.com/watch?v=Kjfzve6lNWI

11. OTHER RELATED ACTIVITIES

S.No	Name of the activity	No. of activities planned	Details
1.	Assignment	3	Details Enclosed
2.	Seminar	-	-
3.	Guest Lectures	-	-
4.	Industrial Visit	-	-

12. CONTENT BEYOND SYLLABUS

1. Anti-Gravity Wheel

13.JOURNALS

IEEE

Equivalent oscillator model for the nonlinear vibration of a porter governor

Author: AkuroBig-AlaboEmmanuel OgheneochukoEkprukeChinwuba VictorOssia

Volume 44, Part 1, 2021, Pages 2895-2901

14. INTERNAL TEST PORTIONS

IAT I -2 Units (Unit 1 and Unit 2)

IAT II - 2 Units (Unit 3 and Unit 4)

MODEL EXAM –5 Units (Unit 1 to Unit5)

STAFF IN-CHARGE **HOD** **ACADEMIC COORDINATOR** **PRINC**

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING & TECHNOLOGY
AMARAVATHIPUDUR, KARAIKUDI-630 301
DEPARTMENT OF MECHANICAL ENGINEERING
ME8594 – DYNAMICS OF MACHINES (R2017)
ASSIGNMENT QUESTIONS

III YEAR/ V SEM **BATCH 2020 -2024**

ASSIGNMENT – 1

S.NO	TOPIC/ QUESTION	DATE OF SUBMISSION
1	Problems based on Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod	
2	Problems based on Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod	
3	Problems based on Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod	
4	Problems based on Turning moment diagrams –Fly Wheels – Flywheels of punching presses	
5	Problems based on Turning moment diagrams –Fly Wheels – Flywheels of punching presses	

ASSIGNMENT – 2

S.NO	TOPIC/ QUESTION	DATE OF SUBMISSION
1	Problems based on Balancing of rotating masses – Balancing a single cylinder engine	
2	Problems based on Balancing of rotating masses – Balancing a single cylinder engine	
3	Problems based on Balancing of rotating masses – Balancing a single cylinder engine	
4	Problems based on Balancing of Multi-cylinder inline, V-engines	
5	Problems based on Balancing of Multi-cylinder inline, V-engines	

ASSIGNMENT – 3

S.NO	TOPIC/ QUESTION	DATE OF SUBMISSION
1	Derivations on Equations of motion.	
2	Derivations on Equations of motion.	
3	Problems on Free vibration.	
4	Problems on Free vibration.	
5	Problems on Forced vibration.	

STAFF INCHARGE **HOD** **DEAN** **ACADEMIC CO-ORDINATOR** **PRINC**

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING & TECHNOLOGY
AMARAVATHIPUDUR, KARAIKUDI-630 301

DEPARTMENT OF MECHANICAL ENGINEERING

ME8594 – DYNAMICS OF MACHINES (R2017)

SEMINAR TOPICS

III YEAR/ V SEM **BATCH 2020 -2024**

S.NO	UNIT	TOPIC	DATE OF SEMINAR
1	1	Fly Wheels	
2	2	Balancing machines.	
3	3	Torsional vibration of shaft	
4	4	Vibration isolation	
5	4	Vibration measurement.	
6	5	Governors – Types	
7	5	Gyroscopic effects in Automobiles	

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING AND TECHNOLOGY KARAIKUDI

for P. Pradeep

STAFF INCHARGE **HOD** **DEAN** **ACADEMIC CO-ORDINATOR** **PRINCIPAL**

PRINCIPAL
Sri Raaja Raajan College of Engg. & Tech
Amaravathipudur, Karaikudi - 630 301
Sivagangai Dist. Tamil Nadu

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ME 8594 – DYNAMICS OF MACHINES

QUESTION BANK

UNIT I - FORCE ANALYSIS

PART-A

1. State the principle of virtual work.
2. Define inertia force and inertia torque.
3. Why flywheels are needed in forging and pressing operations?
4. Draw the turning moment diagram of a single cylinder double acting steam engine.
5. What is free body diagram?
6. Differentiate between static force and inertia force.
7. Differentiate between static and dynamic equilibrium.
8. Define applied and constraint forces.
9. Differentiate between static force analysis and dynamic force analysis.
10. What is the function of flywheel?
11. What is the purpose of flywheel? How does it differ from that of a governor?
12. State D'Alembert's principle.
13. Define piston effort and crank effort.
14. Define crank effort and crank-pin effort.
15. What are the requirements of an equivalent dynamical system?
16. Explain the term maximum fluctuation of energy in fly wheel.
17. Define coefficient of fluctuation of energy.
18. Define coefficient of fluctuation of speed.
19. Explain surge and windup.

PART-B

1. The length of crank and connecting rod of a horizontal reciprocating engine are 100mm and 500 mm respectively. The crank is rotating at 400 rpm. When the crank has turned 30° from the IDC, find analytically, 1. Velocity of piston, 2. Acceleration of piston, 3. Angular velocity of connecting rod, 4. Angular acceleration of connecting rod.

2. A certain machine requires a torque of (5000 + 500 sin θ) N-m to drive it, where θ is the angle of rotation of shaft measured from certain datum. The machine is directly coupled to an engine which produces a torque of (5000 + 600 sin 2θ) N-m. The flywheel and the other rotating parts attached to the engine has a mass of 500 kg at a radius of gyration of 0.4 m. If the mean speed is 150 r.p.m., find:1. the fluctuation of energy, 2. the total percentage fluctuation of speed, and 3. the maximum and minimum angular acceleration of the flywheel and the corresponding shaft position.

The equation of the turning moment curve of a three crank engine is (5000 + 1500 sin 3θ) N-m, where θ is the crank angle in radians. The moment of inertia of the flywheel is 1000 kg-m^2 and the mean speed is 300 r.p.m. Calculate : 1. power of the engine, and 2. the maximum fluctuation of the speed of the flywheel in percentage when (i) the resisting torque is constant, and (ii) the resisting torque is (5000 + 600sinθ) N-m.

4. A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1/2 revolution and remains constant for one revolution, the cycle being repeated thereafter. Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

5. A horizontal steam engine running at 120 r.p.m. has a bore of 250 mm and a stroke of 400 mm. The connecting rod is 0.6 m and mass of the reciprocating parts is 60 kg. When the crank has turned through an angle of 45° from the inner dead centre, the steam pressure on the cover end side is 550 kN/m^2 and that on the crank end side is 70 kN/m^2. Considering the diameter of the piston rod equal to 50 mm, determine: 1. Turning moment on the crank shaft, 2. Thrust on the bearings, and 3. Acceleration of the flywheel, if the power of the engine is 20 kW, mass of the flywheel 60 kg and radius of gyration 0.6 m

6. A single cylinder, single acting, four stroke gas engine develops 25 kW at 320 rpm. The work done by the gases during the expansion stroke is three times the work done on the gases during the compression stroke, the work done during the suction and exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2% of the mean speed and the turning moment diagram during compression and expansion is assumed to be triangular in shape, find the weight of the flywheel if its radius of gyration is 0.5m.

7. The lengths of crank and connecting rod of horizontal steam engine are 300 mm and 1.2 m respectively. When the crank has moved 30° from the inner dead center, the acceleration of piston is 35 m/s^2. The average frictional resistance to the motion of piston is equivalent to a force of 550N and net effective steam pressure on piston is 500 kN/m^2. The diameter of piston is 0.3 m and mass of reciprocating parts is 160 kg. Determine (i) Reaction on the cross-head guides; (ii) Thrust on the crankshaft bearings; and (iii) Torque on the crank shaft.

8. The variation of crankshaft torque of a four cylinder petrol engine may be approximately represented by taking the torque as zero for crank angles 0° and 180° and as 260 Nm for crank angles 20° and 45°, the intermediate portions of the torque graph being straight lines. The cycle is being repeated in every half revolution. The average speed is 600 rpm. Supposing that the engine drives a machine requiring constant torque, determine the mass of the flywheel of radius of gyration 250 mm, which must be provided so that the total variation of speed shall be one percent.

9. A single cylinder vertical engine has a bore of 300 mm and a stroke of 400 mm. The connecting rod is 1 m long and the mass of the reciprocating parts is 140 kg. On the expansion stroke, with the crank at 30° from the top dead center, the gas pressure is 0.7 MPa. If the engine runs at 250 rpm, determine,(i) Net force acting on the piston , (ii) Resultant load on the gudgeon pin, (iii) Thrust on the cylinder walls, and (iv) The speed above which, other things remaining the same, the gudgeon pin load would be reversed in direction.

. In a reciprocating engine mechanism, if the crank and connecting rod are 300 mm and 1 m long respectively and the crank rotates at a constant speed of 200 rpm. Determine analytically,
(i) The crank angle at which the maximum velocity occurs,(ii) Maximum velocity of the piston
(iii)Derive the relevant equations.

11. (i) Deduce the expression for the inertia force in the reciprocating force neglecting the weight of the connecting rod.(ii) A vertical petrol engine with cylinder of 150 mm diameter and 200 mm strokes has a connecting rod of 350 mm long. The mass is 1.6 kg and the engine speed is 1800 rpm. On the expansion stroke with crank angle 30° from TDC, the gas pressure is 750 KPa. Determine the net thrust on the piston.

UNIT-II (BALANCING)

PART-A

1. What is meant by balancing of rotating masses?
2. Why rotating masses are to be dynamically balanced?
3. Define static and dynamic balancing.
4. State the conditions for static and dynamic balancing.
5. Differentiate static and dynamic balancing.
6. State the conditions for complete balance of several masses revolving in different planes of a shaft.
7. Why complete balancing is not possible in reciprocating engine?
8. Can a single cylinder engine be fully balanced? Why?
9. Differentiate between the unbalanced force caused due to rotating and reciprocating masses.
10. Why are the cranks of a locomotive, with two cylinders, placed at 90° to each other?
11. List the effects of partial balancing of locomotives.
12. Define swaying couple.
13. Define hammer blow with respect to locomotives.
14. What are the effects of hammer blow and swaying couple?
15. Define direct and reverse cranks.
16. What for the balancing machines are used?
17. What are different types of balancing machines?
18. Explain the term primary balancing and secondary balancing.
19. What are the conditions required for complete balancing of reciprocating parts?
20. When is a system said to be completely balanced?
21. What is tractive force?

PART-B

1. The following data refer to a two cylinder locomotive with cranks at 90°
   (i) Mass of reciprocating parts/cylinder =300 kg

   (ii) Crank radius =300 mm

   (iii) Diameter of the driving wheel=1800 mm

   (iv) Distance between cylinder axis =600 mm

   (v) Distance between the driving wheel =1600 mm. Determine: (1) The fraction of the reciprocating masses to be balanced if the hammer blow is not to exceed 45 kN at 95 kmph.

   (2) Variation in tractive effort. (3) maximum swaying couple

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

3. A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine :1. The magnitude of the masses at A and D ; 2. the distance between planes A and D ; and 3. the angular position of the mass at D.

4. A five cylinder in-line engine running at 750 r.p.m. has successive cranks 144° apart, the distance between the cylinder centre lines being 375 mm. The piston stroke is 225 mm and the ratio of the connecting rod to the crank is 4. Examine the engine for balance of primary and secondary forces and couples. Find the maximum values of these and the position of the central crank at which these maximum values occur. The reciprocating mass for each cylinder is 15 kg.

5. A, B, C and D are four masses carried by a rotating shaft at radii 100 mm, 125 mm, 200 mm and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the masses of B, C and D are 10kg, 5kg and 4kg respectively. Find the required mass 'A' and relative angular setting of the four masses so that the shaft be in complete balance.

6. A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks. If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force?

7. The following data apply to an outside cylinder uncoupled locomotive :
Mass of rotating parts per cylinder = 350 kg
Mass of reciprocating parts per cylinder = 300 kg
Angle between cranks = 90°
Crank radius = 0.3 m
Cylinder centres = 1.8 m
Radius of balance masses = 0.8 m
Wheel centres = 1.5 m.

If whole of the rotating and 2/3rd of reciprocating parts are to be balanced in planes of the driving wheels, find:1. Magnitude and angular positions of balance masses, 2. Speed in km/hr at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and3. Swaying couple at speed arrived at in (2) above plane.

The axes of the three cylinders air compressor are at 120° to one another, and their connecting rods are coupled to a single crank. The stroke is 160 mm and the length of each connecting rod is 240 mm. The mass of the reciprocating parts per cylinder is 2.4 kg. Find the maximum primary and secondary forces acting on the frame of the compressor when running at 2000 rpm.

9. A shaft carries four rotating masses A, B, C and D in this order along its axis. The mass of B, C and D are 30 kg, 50 kg and 40 kg respectively. The planes containing B and C are 30 cm apart. The angular spacing of the planes containing C and D are 90° and 210° respectively relative to B measured in the same sense. If the shaft and masses are to be in complete dynamic balance, find (i) the mass and the angular position of mass A; and (ii) the position of planes A and D.

10. (i) Four masses m1, m2, m3 and m4 attached to a rotating shaft on the same plane are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if the radius of rotation is 0.2 m.

(ii) Explain with neat sketches, balancing of single revolving mass, by masses in two different planes in a rotating system.

11. A shaft carries four rotating masses A, B, C and D which are completely balanced. The masses B, C and D are 50 kg, 80 kg and 70 kg respectively. The masses C and D make angles of 90° and 195° respectively with mass B in the same sense. The masses A, B, C and D are concentrated at radius 75 mm, 100 mm, 50 mm and 90 mm respectively. The plane of rotation of masses B and C are 250 mm apart. Determine (i) the magnitude of mass A and its angular position (ii) the position of planes A and D.

12. A shaft is rotating at a uniform angular speed. Four masses M1, M2, M3 and M4 of magnitudes 300 kg, 450 kg, 360 kg, 390 kg respectively are attached rigidly to the shaft. The masses are rotating in the same plane. The corresponding radii of rotation are 200 mm, 150 mm, 250 mm and 300 mm respectively. The angles made by these masses with horizontal are 0°, 45°, 120° and 255° respectively. Find, (i) the magnitude of balancing mass,(ii) the position of balancing mass if its radius of rotation is 200mm.

13. A four cylinder vertical engine has cranks 225mm long. The plane of rotation of the first, second and fourth cranks are 600 mm,300 mm and 300mm respectively from that of the third crank and their reciprocating masses are 100 kg, 120 kg and 100kg respectively. Find the mass of the reciprocating parts for the third cylinder and relative angular position of the cranks in order that the engine may be in complete primary balance.

UNIT-III FREE VIBRATION

PART-A

1. Write the vibration characteristics.
2. What is the difference between viscous damping and coulomb damping?
3. What are the causes and effect of vibration?
4. Define frequency, cycle, period and free vibration.
5. What are the different types of vibrations?
6. State different method of finding natural frequency of a system.
7. What is meant by degrees of freedom in a vibrating system?
8. What is the natural frequency of simple spring mass system?

. Determine the natural frequency of mass of 10 kg suspended at the bottom of two springs of stiffness: 5N/mm and 8N/mm in series.

10. What is the effect of inertia on the shaft in longitudinal and transverse vibrations?
11. What are the different types of damping?
12. Draw the schematic diagram of a free damped vibration system and write the governing differential equation of the system.
13. Sketch the Time Vs Displacement for under-damped and over-damped systems.
14. What is the limit beyond which damping is detrimental and why?
15. What is meant by critical damping?
16. What type of motion is exhibited by a vibrating system when it is critically damped?
17. Define critical or whirling speed.
18. What are the factors that affect the critical speed of a shaft?
19. What are the causes of critical speed?

PART-B

1. A vertical steel shaft 15 mm diameter is held in long bearings 1 m apart and carries at its middle a disc of mass 15 kg. The eccentricity of the centre of gravity of the disc from the centre of the rotor is 0.30mm. The modulus of elasticity for the shaft material is 200 GN/m^2 and the permissible stress is 70 MN/m^2. Determine: 1. The critical speed of the shaft and (ii).The range of speed over which it is unsafe to run the shaft. Neglect the mass of the shaft.

2. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E = 200 GN/m^2.

3. A steel shaft ABCD 1.5m long has flywheel at its ends A and D. The mass of the flywheel A is 600 kg and has a radius of gyration of 0.6 m. the mass of the flywheel D is 800 kg and has a radius of gyration of 0.9m. the connecting shaft has a diameter of 50 mm for the portion AB which is 0.4 m long; and has a diameter of 60 mm for the portion BC which is 0.5 m long; and has a diameter of d mm for the portion BC which is 0.6 m long. Determine :(i) The diameter 'd' of the portion CD so that the node of the torsional vibration of the system will be at the centre of the length BC; and (ii) the natural frequency of the torsional vibrations. The modulus of rigidity of the shaft material is 80GN/m^2.
4. Determine: (i) the critical damping co-efficient, (ii) the damping factor, (iii) the natural frequency of damped vibrations, (iv) the logarithmic decrement and (v) the ratio of two consecutive amplitudes of a vibrating system which consists of a mass of 25 kg, a spring of stiffness 15 kN/m and a damper. The damping provided is only 15% of the critical value.

5. A shaft of length 1.25 m is 75 mm in diameter for the first 275 mm of length, 125 mm in diameter for the next 500 mm length, 87.5 mm in diameter for the next 375 mm length and 175 mm in diameter for the remaining 100 mm of its length. The shaft carries two rotors at two ends. The mass moment of inertia of the first rotor is 75kgm^2 whereas of the second rotor is 50 $kg\text{-}m^2$. Find the frequency of natural torsional vibrations of the system. The modulus of the rigidity of the shaft material may be taken as 80 GN/m^2.

6. Derive an expression for the natural frequency of the free longitudinal vibration by (i)Equilibrium method (ii) Energy method (iii)Rayleigh's method

7. Between a solid mass of 10 kg and the floor are kept two slabs of isolators, natural rubber and felt, in series. The natural rubber slab has a stiffness of 3000 N/m and an equivalent viscous damping coefficient of 100 N.sec/m. The felt slab has a stiffness of 12000 N/m and equivalent viscous damping coefficient of 330 N.sec/m. Determine the un-damped and the damped natural frequencies of the system in vertical direction, neglecting the mass of isolators.

8. A steel shaft 1.5 m long is 95 mm in diameter for the first 0.6 m of its length, 60 mm in diameter for the next 0.5 m of the length and 50 mm in diameter for the remaining 0.4 m of its length. The shaft carries two flywheels at two ends, the first having a mass of 900 kg and 0.85 m radius of gyration located at the 95 mm diameter end and the second having mass of 700 kg and 0.55 m radius of gyration located at the other end. Determine the location of the node and the natural frequency of free torsional vibration of the system. The modulus of rigidity of the shaft material may be taken as 80 GN/m^2.

9. A vibrating system consists of a mass of 8 kg, spring of stiffness 5.6 N/mm and dashpot of damping coefficient of 40 N/m/s. Find, (i) Critical damping coefficient (ii) the damping factor (iii) the natural frequency of damped vibration (iv) the logarithmic decrement (v) the ratio of two consecutive amplitude (vi) the number of cycle after which the original amplitude is reduced to 20 percent.

10. The barrel of a large gun recoils against a spring on firing. At the end of the firing, a dashpot is engaged that allows the barrel to return to its original position in minimum time without oscillation. Gun barrel mass is 400 kg and initial velocity of recoils 1m. Determine spring stuffiness and critical damping coefficient of dashpot.

11. In a single degree of damped vibration system a suspended mass of 8 kg makes 30 oscillations in 18 seconds. The amplitude decreases to 0.25 of the initial value after 5 oscillations. Determine (i) the spring stiffness (ii) logarithmic decrement (iii) damping factor (iv) Damping coefficient.

UNIT-IV (FORCED VIBRATION)

PART-A

1. What is fundamental frequency?
2. What is meant by forced vibration?
3. Define damping ratio or damping factor.
4. Define logarithmic decrement.
5. Define resonance.
6. Give equation for damping factor ζ and damped frequency ω_d.
7. What is meant by harmonic forcing?
8. What is meant by dynamic magnifier or magnification factor?
9. Define transmissibility.
10. Define transmissibility ratio or isolation factor.
11. What is vibration isolation?

12. Write down the expression for amplitude of forced vibration.
13. List out the sources of excitation in forced vibration.
14. Show that for effective isolation of vibration, frequency ratio $r > \sqrt{2}$.
15. Define step input and harmonic forcing function.

PART-B

1. A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decreases to one- tenth of the initial value in four complete oscillations. If a periodic force of 150cos50t N is applied at the mass in the vertical direction .Find the amplitude of the forced vibrations? What is its value of resonance?

2. A machine part of mass 2 kg vibrates in a viscous medium. Determine the damping coefficient when a harmonic exciting force of 25 N results in resonant amplitude of 12.5 mm with a period of0.2 second. If the system is excited by a harmonic force of frequency 4 Hz, what will be the percentage increase in the amplitude of vibration when damper is removed as compared with that with damping?

3. A machine supported symmetrically on four springs has a mass of 80 kg. The mass of the reciprocating parts is 2.2 kg which move through a vertical stroke of 100 mm with simple harmonic motion. Neglecting damping, determine the combined stiffness of the spring so that the force transmitted to foundation is $1/20^{th}$ of the impresses force. The machine crank shaft rotates at 800 rpm. If under working conditions, the damping reduces the amplitudes of successive vibrations by 30%, find (i) the force transmitted to the foundation at resonance and (ii) the amplitude of vibration at resonance.

4. A harmonic exiting force of 25N is acting on a machine part which is having a mass of 2 Kg and vibrating in viscous medium. The exciting force causes resonant amplitude of 12.5 mm with a period of 0.2 sec. Determine damping coefficient.

5. A mass of 50 kg is supported by an elastic structure of total stiffness 20 KN/m. The damping ratio of the system is 0.2. A simple harmonic disturbing force acts on the mass and at any time't' seconds, the force is 60sin10t newtons. Find amplitude of the vibration and phase angle caused by the damping.

6. A mass of 50 kg is supported by an elastic structure of total stiffness 20 KN/m. The damping ratio of the system is 0.25. A simple harmonic disturbing force acts on the mass and at any time't' seconds, the force is 75cos12t newtons. Find amplitude of the vibration and phase angle caused by the damping.

7. A body having a mass of 15 kg is suspended from a spring which deflects 12 mm under the weight of the mass. Determine the frequency of the free vibrations. What is the viscous damping force needed to make the motion a periodic at a speed of 1 mm/s? If, when damped to this extend a disturbing force having a maximum value of 100 N and vibrating at 6 Hz is made to act on the body, determine the amplitude of the ultimate motion.

8. A single cylinder vertical petrol engine of total mass of 200 kg is mounted upon a steel chassis frame. The vertical static deflection of the frame is 2.4 mm due to the weight of the engine .The mass of the reciprocating parts is 18 kg and stroke of piston 160mm with S.H.M. If dashpot of damping coefficient of 1 N/mm/s used to damped the vibrations, calculate al steady state
(i) Amplitude of vibrations at 500 rpm engine speed. (ii)The speed of the driving shaft at which resonance will occurs.

A single cylinder engine has an out of balance force of 500 N at an engine speed of 30rpm.The total mass of engine is 150 kg and its carried on a set of total stiffness 300N/cm.(i) Find the amplitude of steady motion of the mass and maximum oscillating force transmitted to the foundation. (ii)If a viscous damping is interposed between the mass and the foundation the damping force 1000 N at 1 m/s of velocity, find the amplitude of force damped oscillation of the mass and its angle of lag with disturbing force.

. The mass of an electric motor is 120kg and it runs at 1500rpm.The armature mass is 35kg and its C.G. lies 0.5mm from axis of rotation. The motor is mounted on five springs of negligible damping. So that the force transmitted is one-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the five springs. Determine

(i) The stiffness of the spring (ii) the dynamic force transmitted to the base at the operating speed. (iii) Natural frequency of system.

UNIT-V (GOVERNOR AND GYROSCOPE)

PART-A

1. What is the function of governor?
2. How governors are classified?
3. Differentiate between governors and fly wheel.
4. What is meant by applied torque and reaction torque?
5. What is meant by sensitiveness of a governor?
6. What is the effect of friction on the governor?
7. What is meant by hunting?
8. What is meant by isochronous conditions governor?
9. Differentiate between isochronous governor and sensitiveness of a governor.
10. Define the meaning of stability of a governor.
11. What is gyroscopic torque?
12. What is the effect of gyroscopic couple on rolling of ship? Why?
13. Define gyroscope.
14. What is meant by reactive gyroscopic couple?
15. Define gyroscopic couple.
16. Write expression for gyroscopic couple.
17. What is the effect of gyroscopic couple on an automobile taking a turn?
18. Distinguish between pitching and rolling motion of ships.
19. What will be the effect of gyroscopic couple on a disc fixed at a certain angle to a rotating shaft?
20. List some of the terms related to motion of ships using gyroscopic principle.

PART-B

1. A spring loaded governor of the Hartnell type has equal arms. The balls rotate in a circle of 15 cm dia when the sleeve is in the mid position and the ball arms are vertical. The equilibrium speed for this position is 500 rpm. The maximum sleeve movement is to be 3 cm and the maximum variation of speed taking in account the friction to be of the mid position speed. The mass of the sleeve is 5 kg and the friction force may be consideration to arise out of an equivalent to 3 kg at the sleeve. The power of the governor must be sufficient to overcome the friction by ±1% change of speed either way at mid-position. Determine 1. The rotating mass; 2. the spring stiffness 3. the initial compression of spring. Neglect obliquity effect of arms.

2. The driving axle of a locomotive with two wheels has a mass moment of inertia of 350 kg m^2. The wheels are 1.8 min. diameter. The distance between the planes of the wheels is 1.5m. When travelling at 100 km/hr the locomotive passes over a defective rail which causes the right hand wheel to fall 10 mm and rise again in a total time of 0.1 sec, the vertical movement of the wheel being with SHM. Find the maximum gyroscopic torque caused. Determine the direction in which it acts when the wheel is failing. Let the linear motion of the right hand wheel be $\alpha \cos qt$ where α=0.005m and $q=2\pi/0.1$ rad/sec.

3. In an engine governor of the Porter type, the upper and lower arms are 200mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, Find, taking friction into account, range of speed of the governor.

4. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle neglecting a curve.

5. The radius of rotation of the balls of a Hartnell governor is 80 mm at the minimum speed of 300 r.p.m. Neglecting gravity effect, determine the speed after the sleeve has lifted by 60 mm. Also determine the initial compression of the spring, the governor effort and the power. The particulars of the governor are given below: Length of ball arm = 150mm; length of sleeve arm = 100mm; mass of each ball = 4 kg; and stiffness of the spring = 25 N/mm.

6. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship: 1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h. 2. When the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

7. A porter governor has equal arms each 250mm long and pivoted on the axis of rotation. Each ball has a mass of 5kg and mass of the central load on the sleeve is 25kg.The radius of rotation of the ball is 150mm when governor begins to lift and 200mm when the governor is at maximum speed. Find the maximum and minimum speed and range of speed of the governor.

8. Calculate the rage of speed of a porter governor which has equal arms of each 200mm long and pivoted on the axis of rotation .The mass of each ball is 4kg and the central load of the sleeve is 20kg.The radius of rotation of the ball is 100mm when the governor being to lift and 130mm when the governor is at maximum speed.

A porter governor has equal arms each 250mm long. The upper arms are attached on the axis of rotation and the lower are attached to sleeve at a distance of 30mm from the axis.. Each ball has a mass of 5kg and mass of the central load on the sleeve is 50kg.The radius of rotation of the ball is 150mm when governor begins to lift and 200mm when the governor is at maximum speed. Find the maximum and minimum speed and range of speed of the governor.

10. A porter governor has equal arms each 250mm long. The upper arms are attached to sleeve at a distance of 40mm and the lower are attached to sleeve at a distance of 50mm from the axis.. Each ball has a mass of 2.5kg and mass of the central load on the sleeve is 25kg. The force of friction on the sleeve is 20N. The radius of rotation of the ball is 125mm when governor begins to lift and 150mm when the governor is at maximum speed. Find the maximum and minimum speed and range of speed of the governor.

11. A hartnell governor having a central sleeve spring and two right angled bell crank lever operates between 290rpm and 310rpm for a sleeve lift of 15mm.The sleeve and ball arms are 80mm and 120mm respectively. The levers are pivoted at 120mm from the governor axis and mass of the ball is 2.5kg.The ball arms are parallel at lowest equilibrium speed. Determine (i) load on the spring at maximum and minimum speeds and (ii) Stiffness of the spring.

12. The controlling force in a spring controlled governor is 1500N when radius of rotation is 200mm and 887.5N when radius of rotation is 130mm.The mass of each ball is 8kg.If the controlling force curve is a straight line, then find (i) Controlling force at 150mm radius of rotation (ii) Speed of the governor at 150mm radius.(iii)Increase in initial tension so that governor is isochronous. (iv) Isochronous speed.

13. In a spring controlled governor, the controlling force curve is a straight line. When the balls are 400mm apart, the controlling force is 1200N and when 200mm apart, the controlling force is 450N.Determine the speed at which the governor runs when the balls are 250mm apart. When initial tension on the spring would be required for isochronisms and what would be the speed. Take mass of each ball to be 10kg.

14. Calculate the range of speed of a proell governor, which has equal arms each of 300mm and are provided on the axis of rotation. The mass of each ball is 10kg and the central mass on the sleeve is 100kg.The extension arms of the lower links are each 80mm long and parallel to the axis when the radius of rotation of the balls are 150mm and 200mm.

15. The rotor of a turbine yatch rotates at 1200rpm clockwise when viewed from stern. The rotor has a mass of 750 kg and radius of gyration of 250mm.Find the maximum gyroscopic couple transmitted to the hull when yacht pitches with a maximum angular velocity of 1 rad/s. What is the effect of this couple?

16. Each paddle wheel of a steamer have a mass of 1600kg and a radius of gyration of 1.2meters.The steamer turns to port in a circle of 160meters radius at 24Km/hr.The speed of the paddle is90rpm.Find the magnitude and effect of the gyroscopic couple acting on the steamer. (16)

17.The turbine rotor of a ship has a mass of 20 tones and a radius of gyration 0.75.Its speedis 2000rpm.The ship pitches 6° above and below the horizontal position .One complete oscillation takes 18 seconds and the motion is simple harmonic. Determine (i) the maximum couple tending to. shear the holding down bolt of the turbine. (ii)The maximum angular acceleration of the ship during pitching(iii) The direction in which the bow will tend to turn while, if the rotation of the rotor is clockwise when locking from rear.

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth/Fifth Semester

Mechanical Engineering

ME 8594 — DYNAMICS OF MACHINES

(Common to Mechanical Engineering (Sandwich)/Mechatronics Engineering)

(Regulations 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. List the two conditions for static equilibrium.

2. State D'Alembert's principle.

3. Mention any three factors that contribute to unbalance in a rotating system.

4. When is a system said to be completely balanced?

5. What do you understand by degrees of freedom?

6. List the various types of dampers.

7. Calculate the deflection of the spring of stiffness 35 kN to which a body of mass 35 kg is suspended.

8. Define transmissibility.

9. What do you mean by sensitiveness of governor?

10. What is gyroscopic torque?

PART B — (5 × 13 = 65 Marks)

11. (a) A vertical double acting steam engine has a cylinder 300 mm in diameter, 450 mm stroke and operates at 200 rpm. The mass of reciprocating parts is 225 kg, the piston rod is 50 mm in diameter and the connecting rod is 1 metre long. When crank has turned through 125° from the top dead centre the steam pressure above the piston is 30 kN/m² and below the piston is 2 kN/m². Calculate the tangential force on the crank pin and effective turning moment on the crankshaft. (13)

Or

(b) The turning moment diagram for engine consist of two isosceles triangles with maximum height of each triangle representing turning moment equal to 1000 Nm, while the base of the triangle is from $\theta = 0°$ to 180° $\theta = 180°$ to 360°. If the engine runs at 200 rpm and if the total fluctuation of speed is not to exceed 2.8%, calculate the indicated horsepower of the engine and moment of inertia of the flywheel. (13)

12. (a) Shaft has eccentrics each 7.5 cm diameter and 2.5 cm thick machined in one piece with the shaft. The central planes of the eccentrics are 6 cm apart. The distance of the centres from the axis of rotation are 1.2 cm, 1.9 cm and 1.2 cm with angular positions 120° apart and the metal weights 0.007 kg/m³. Calculate the out of balance force and couple at 600 rpm. If the shaft is balanced by adding two weights of radius 7.5 cm and at a distance of 10 cm from the central plane of the middle eccentrics, find the weights to be added and their angular positions. (13)

Or

(b) An air compressor has four inline cylinders at 90° intervals. The crank radius is 140 mm, while the connecting rod is 560 mm long for each cylinder. The mass of reciprocating parts is 20 kg for each cylinder and speed of rotation is 600 rpm and the cylinders are 300 mm apart. Prove that there is nil out of balance primary and secondary forces. Determine the corresponding magnitude of primary and secondary couples. (13)

13. (a) A vibration system consists of a mass 60 kg, a spring of stiffness 30 kN/m and a damper. The damping provided is only 20% of the critical value. Determine (13)

(i) the damping factor,

(ii) critical damping coefficient,

(iii) natural frequency of the damped vibration

(iv) log logarithm decrement and

(v) ratio of the two consecutive amplitudes.

Or

(b) The flywheel of an engine driving a Dynamo has a mass of 180 kg and radius of gyration of 25 mm. The shaft at the flywheel end has an effective length of 250 mm and 50 mm diameter. The mass of the armature is 120 kg and its radius of gyration is 22.5 mm. The Dynamo shaft has 50 mm diameter and 200 mm effective length. Calculate the position of the node and frequency of torsional oscillation. Assume $C = 83$ kM/mm^2. (13)

14. (a) A motor of weight 25 kg is mounted on a rubber pad which deflects by 1 mm due to motor weight. The rotor weighs 5 kg and has an eccentricity of 0.1 mm and rotates at 1500 rpm. Find the amplitude of vibration of the motor and the force transmitted to the foundation under no damping and a damping factor of 0.1 condition. (13)

Or

(b) Determine the displacement amplitude of 1.5 Ton car moving with velocity of 100 kmph as it passes over a rough road which has a sinusoidal surface with amplitude of 75 mm and wavelength of 5 mm. The suspension system has spring constant of 500 N/mm and damping ratio of 0.5. (13)

15. (a) A porter governor has all four arms 300 mm long. The upper arms are pivoted on the axis of rotation. Each ball has a mass of 4 kg and mass of central load on the sleeve is 50 kg. The radius of rotation of the balls is 200 mm if the lower arms are attached to the sleeve at 35 mm from the axis, determine the equilibrium speed of the governor. (13)

Or

(b) The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 rpm clockwise when looking from a stern. The radius of gyration of the rotor is 0.5 m. Determine the gyroscope couple and its effect on the ship when the ship is steering to the right in a curve of 100 m radius at a speed of 29000 m/hr. Also, calculate the torque and its effect when the ship is pitching in Simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 50 seconds and the angular displacement between the two extreme positions of the pitching is 12°. Find the maximum acceleration during pitching motion. (13)

PART C — (1 × 15 = 15 marks)

16. (a) Investigate the term involved in the equation of motion of one degree of freedom system as given by $5\ddot{x} + 3\dot{x} + 12x = 10 \sin \omega t$. (15)

Or

(b) Prove that the resultant unbalanced force is minimum, when half of the reciprocating masses are balanced by rotating masses i.e. when c = 1/2. (15)

SRI RAAJA RAAJAN COLLEGE OF ENGINEERING AND TECHNOLOGY,

AMARAVATHIPUDUR

DEPARTMENT OF MECHANICAL ENGINEERING

INTERNAL ASSESSMENT - I

ME8594 - DYNAMICS OF MACHINES

IIIRD YEAR/ V[TH] SEM

DURATION: 2 HRS

MAX MARKS: 100

PART – A (5 X 2 Marks)

Answer all the questions

State D'Alembert's principle.

Define coefficient fluctuation of energy.

Define the coefficient of fluctuation of speed.

Write the equation for inertia force due to reciprocating parts in IC engine.

What is meant by crank pin effort ?

What is meant by balancing of a single rotating mass ?

Differentiate static and dynamic balancing.

Define static balancing of shaft.

State the reasons for choosing multi-cylinder engine in comparison with that of the single cylinder engine.

What are primary and secondary unbalanced forces ?

PART B (4 X 10 Marks)

Answer all the questions

The lengths of crank and connecting rod of horizontal steam engine are 300 mm and 1.2 m respectively. When the crank has moved 30° from the inner dead center, the acceleration of piston is 35 m/s². The average frictional resistance to the motion of piston is equivalent to a force of 500 N and net effective steam pressure on piston is 500 kN/m². The diameter of the piston is 0.3 m and mass of reciprocating parts is 160 kg. Determine

- Reaction on the cross-head guides
- Thrust on crank shaft bearing
- Torque on the crank shaft.

The crank-pin circle radius of a horizontal engine is 300mm. The mass of the reciprocating parts is 250 kg. When the crank has travelled 60° from I.D.C., the difference between the driving and the back pressures is $0.35\,N/mm^2$. The connecting rod length between centres is 1.2m and the cylinder bore is 0.5m. If the engine runs at 250 r.p.m., and if the effect of piston rod diameter is neglected, calculate the pressure on slide bars, the thrust in the connecting rod, the tangential force on the crank-pin and the turning moment on the crank shaft.

The turning moment diagram for a multi-cylinder engine has been drawn to a vertical scale of 1mm = 650 Nm and a horizontal scale of 1 mm = 4.5°. The areas above and below the mean torque line are -28, $+380$, -260, $+310$, -300, $+242$, -380, $+265$ and $-229\,mm^2$. The fluctuation of speed is limited to ± 1.8% of the mean speed which is 400 rpm. The density of the rim material is 7000 kg/m³ and the width of the rim is 4.5 times its thickness. The centrifugal stress in the rim material is limited to 6 N/mm². Neglecting the effect of the boss, and the arms, determine the diameter and the cross-section of the flywheel rim.

The turning moment diagram for a four stroke gas engine is represented by four triangles. The area form line of zero pressure are as follows : 1. Suction stroke 350 mm² 2. Compression stroke 1400 mm² 3. Expansion stroke 3550 mm² 4. Exhaust stroke 500 mm². Each mm² represents 3N-m. Assuming the resisting moment to be uniform, find the mass of rim of flywheel required to keep the mean speed as 200 rpm within ±2%. The mean radius of the rim may be taken as 0.75 m. Also determine the crank position for maximum and minimum speeds.

A circular disc mounted on a shaft carries three attached masses of 4 kg, 3 kg, and 2.5 kg at radial distances of 75 mm, 85 mm, and 50 mm, and at angular positions of 45°, 135° and 240° respectively. The angular positions are measured counter - clockwise from the reference line along x-axis. Determine the amount of the counter-mass at a radial distance of 75 mm required for the static balance.

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 200 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Three masses are attached to a shaft as follows: 10 kg at 90 mm radius, 15 kg at 120 mm radius and 8 kg at 150 mm radius. The masses are to be arranged so that shaft is in static balance. Determine the angular position of masses relative to 10 kg mass. All the masses are in the same plane.

A shaft carries four masses in parallel planes A, B, C, and D in this order along its length. The masses at B and C are 20 kg and 15 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine the magnitude of the masses at A and D, the distance between the planes A and D and the angular position of the mass at D.

The ratio of connecting rod length to crank length of a vertical gasoline engine is 4. The engine bore and stroke is 8 cm and 10 cm respectively. The mass of reciprocating parts is 1 kg. The gas pressure on the piston is 6 bar, when it has moved 40° from the inner dead centre on its power stroke. Determine

- Net Load on the piston
- Net load on gudgeon pin and the crank piston
- Thrust on the cylinder walls
- Thrust on crank bearing

The engine runs at 2000 rpm. At what engine speed will this load on gudgeon pin or the crank pin will be zero ?

The turning moment diagram for a multi cylinder engine has been drawn to a scale of 1 mm = 4500 N-m vertically and 1 mm = 2.4° horizontally. The intercepts between output and mean resistance line taken in order from one end are 342, 230, 245, 303, 115, 232, 227 and 164 mm^2 and the engine runs at 150 rpm. If the mass of the flywheel is 1000 kg and the total fluctuation of speed does not exceed 3% of mean speed, find the radius of gyration.

PART – C (1 x 15 Marks)

i) Differentiate between static and dynamic balancing.

ii) A circular disc mounted on a shaft carries three attached masses 4kg, 3 kg and 2.5 kg at radial distances 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively. The angular positions are measured counter-clockwise from the reference line along x-axis. Determine the amount of the counter mass at a radial distance of 75 mm required for the static balance.

An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

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DEPARTMENT OF MECHANICAL ENGINEERING
INTERNAL ASSESSMENT -2
ME8594 – DYNAMICS OF MACHINES

IIIRD YEAR/ VTH SEM

DURATION: 1.5 HRS
MAX. MARKS: 50

PART – A (5 X 2 Marks)
Answer all the questions

Define logarithmic decrement.

State different methods of finding natural frequency of a system.

Differentiate node and antinode with respect torsional vibrations.

What is damping?

What are the different types of damped vibrations ?

What is vibration isolation ?

Define transmissibility.

List the types of external excitations.

What are the instruments used for measurement of vibration levels in a body ?

Define dynamic magnifier.

PART B (4 X 10 Marks)
Answer all the questions

A machine mounted on springs and fitted with a dashpot has a mass 60 kg. There are three springs in parallel each of stiffness 12 N/mm. The amplitude of vibration reduces from 45 to 8 mm in two complete oscillations. Assume that the damping force varies as the velocity. Determine (a) damping coefficient (b) the ratio of frequencies of damped and undamped vibrations (c) the periodic time of damped vibration.

A shaft 1.5 m long, supported in flexible bearings at the ends carries two wheels each of 50 kg mass. One wheel is situated at the centre of the shaft and the other at a distance of 375 mm from the centre towards left. The shaft is hollow of external diameter 75 mm and internal diameter 40 mm. The density of the shaft material is 7700 kg/m^3 and its modulus of elasticity is 200 GN/m^2. Find the lowest whirling speed of the shaft, taking into account the mass of the shaft.

A coil of spring stiffness 4 N/mm supports vertically a mass of 20 kg at the free end. The motion is resisted by the oil dashpot. It is found that the amplitude at the beginning of the fourth cycle is 0.8 times the amplitude of the previous vibration. Determine the damping force per unit velocity. Also find the ratio of the frequency of damped and undamped vibrations.

In a damped free vibrations mass is 2 kg and Spring stiffness is 100 N/m. It is observed that an initial amplitude of 100 mm is reduced to 1 mm in 10 oscillations. Find the damping constant and the natural frequency of vibrations.

A centrifugal fan of mass 5 kg has a rotating unbalance of 0.25 kg m. When dampers having damping factor of 0.2 are used, specify the springs for mounting such that only 10% of the unbalance force is transmitted to the floor. The fan is running at a constant speed of 1000 rpm.

A vehicle of mass 1200 kg is travelling on a road, the surface of which varies sinusoidally with an amplitude of 0.05 m and wave length of 6 m. The suspension system has a spring constant of 400 kN/m and a damping factor of 0.5. If the vehicle speed is 100 km/hr, determine the displacement amplitude of the vehicle.

A 1000 kg machine is mounted on four identical springs of total spring constant K and having negligible damping. The machine is subjected to a harmonic external force of amplitude $F_0 = 490$ N and frequency 180 rpm. Determine the amplitude of motion of the machine and the maximum force transmitted to the foundation because of the unbalanced force when $K = 1.96 \times 10^6$ N/m.

The springs of an automobile trailer are compressed 0.1m under its own weight. Find the critical speed when the trailer is travelling over a road with a profile approximated by a sine wave of amplitude 0.08 m and wave length of 14 m. What will be the amplitude of vibration at 60 km/hr?

A single-cylinder vertical petrol engine of total mass of 200 kg is mounted upon a steel chassis frame. The vertical static deflection of the frame is 2.4 mm due to the weight of the engine. The mass of the reciprocating parts is 18 kg and the stroke of the piston is 160 mm with SHM. If dashpot of damping coefficient of 1 N/mm/s is used to dampen the vibrations, calculate at steady state, the amplitude of forced vibrations at 500 rpm engine speed and the speed of driving shaft at which resonance will occur.

A compressor supported symmetrically on four springs has a mass of 100 kg. The mass of the reciprocating parts is 2 kg which move through a vertical stroke of 80 mm with SHM. Neglecting damping, determine the combined stiffness of the springs so that the force transmitted to the foundation is $1/25^{th}$ of the impressed force. The machine crankshaft rotates at 1000 rpm. When the compressor is actually supported on springs, it is found that the damping reduces the amplitude of successive free vibrations by 25%. Find the force transmitted to the foundations at 1000 rpm, the force transmitted to the foundation at resonance and the amplitude of the vibrations at resonance.

PART – C (1 x 15 Marks)

A steel shaft ABCD 1.5 m long has flywheel at its ends A and B. The mass of flywheel A is 500 kg and radius of gyration 0.6 m. The mass of the flywheel D is 700 kg and has a radius of gyration 0.9 m. The connecting shaft has a diameter 60 mm for the portion AB which is 0.4 m long and has a diameter of 70 mm for BC which is 0.5 m long and has a diameter of d for the portion CD which is 0.6 m long. Determine i) the diameter of portion CD so that the node of the torsional vibration of the system will be at the centre of length BC and ii) Natural frequency of the torsional vibrations. The modulus of rigidity for the shaft material is 80 GN/m^2.

The following data relate to a shaft held in long bearings.

Length of the shaft – 1.2 m

Diameter of the shaft – 14 mm

Mass of a rotor at midpoint – 16 kg

Eccentricity of centre of mass of rotor from centre of rotor – 0.4 mm

Modulus of Elasticity of shaft material – 200 GN/mm^2

Permissible stress in shaft material – 70×10^6 N/m^2

Determine the critical speed of the shaft and the range of speed over which it is unsafe to run the shaft. Assume the shaft to be mass less.

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DEPARTMENT OF MECHANICAL ENGINEERING

MODEL EXAMINATION

ME8594 – DYNAMICS OF MACHINES

IIIRD YEAR/ V[TH] SEM

DURATION: 3 HRS

PART – A (10 X 2 Marks)
Answer all the questions

1 Define the coefficient of fluctuation of speed.

2 Distinguish between crank effort and piston effort.

3 What is meant by crank-pin effort?

4 State D'Alembert's principle.

5 Differentiate transverse and torsional vibrations.

6 Define logarithmic decrement, using a neat sketch.

7 Give the general equation of forced vibration of a mechanical system.

8 List the requirements of Isolators.

9 Define governor effort.

10 Sketch a gyroscope and indicate the parts.

PART B (5 X 13 Marks)

11. a) The lengths of crank and connecting rod of a horizontal engine are 200 mm and 1 m respectively. The crank is rotating at 400 rpm. When the crank has turned through 30° from the inner dead centre, the difference of pressure between cover and piston rod 0.4 N/mm^2. If the mass of the reciprocating parts is 100 kg cylinder bore is 0.4 m, then calculate, the inertia force, force on piston, piston effort, thrust on the sides of the cylinder walls, the thrust in the connecting rod, and the crank effort.

(OR)

b) The turning moment diagram for a four stroke gas engine is represented by four triangles. The area form line of zero pressure are as follows : 1. Suction stroke 350 mm^2 2. Compression stroke 1400 mm^2 3. Expansion stroke 3550 mm^2 4. Exhaust stroke 500 mm^2. Each mm^2 represents 3N-m. Assuming the resisting moment to be uniform, find the mass of rim of flywheel required to keep the mean speed as 200 rpm within ±2%. The mean radius of the rim may be taken as 0.75 m. Also determine the crank position for maximum and minimum speeds.

12. a) A shaft carries four masses in parallel planes A, B, C, and D in this order along its length. The masses at B and C are 20 kg and 15 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine the magnitude of the masses at A and D, the distance between the planes A and D and the angular position of the mass at D.

(OR)

b) An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

13. (a) In a damped free vibrations mass is 2 kg and Spring stiffness is 100 N/m. It is observed that an initial amplitude of 100 mm is reduced to 1 mm in 10 oscillations. Find the damping constant and the natural frequency of vibrations.

Or

b) The following data relate to a shaft held in long bearings.

Length of the shaft – 1.2 m

Diameter of the shaft – 14 mm

Mass of a rotor at midpoint – 16 kg

Eccentricity of centre of mass of rotor from centre of rotor – 0.4 mm

Modulus of Elasticity of shaft material – 200 GN/mm^2

Permissible stress in shaft material – 70×10^6 N/m^2

Determine the critical speed of the shaft and the range of speed over which it is unsafe to run the shaft. Assume the shaft to be mass less.

14. (a) A 1000 kg machine is mounted on four identical springs of total spring constant K and having negligible damping. The machine is subjected to a harmonic external force of amplitude $F_0 = 490$ N and frequency 180 rpm. Determine the amplitude of motion of the machine and the maximum force transmitted to the foundation because of the unbalanced force when $K = 1.96 \times 10^6$ N/m.

Or

b) A compressor supported symmetrically on four springs has a mass of 100 kg. The mass of the reciprocating parts is 2 kg which move through a vertical stroke of 80 mm with SHM. Neglecting damping, determine the combined stiffness of the springs so that the force transmitted to the foundation is $1/25^{th}$ of the impressed force. The machine crankshaft rotates at 1000 rpm. When the compressor is actually supported on springs, it is found that the damping reduces the amplitude of successive free vibrations by 25%. Find the force transmitted to the foundations at 1000 rpm, the force transmitted to the foundation at resonance and the amplitude of the vibrations at resonance.

15. a) The arms of a Porter governor are 250 mm long. The upper arms are pivoted on the axis of revolution, but the lower arms are attached to a sleeve at a distance of 50 mm from the axis of rotation. The weight on the sleeve is 600 N and the weight of each ball is 80 N. Determine the equilibrium speed when the radius of rotation of the balls is 150 mm. If the friction is equivalent to a load of 25 N at the sleeve, determine the range of speed for this position.

(OR)

b) In a spring loaded governor of the Hartnell type, the weight of each ball is 50 N and the sleeve lift is 50 mm. The governor begins to float at 240 rpm when its radius is 110 mm. The mean speed of the governor is 20 times the range of speed when the friction is neglected. The lengths of the ball and roller arms are 12 and 10 cms. Distance of the pivot of bell crank lever from governor axis is 14 cm. Find the initial compression on the spring, taking into account of the obliquity of the arms.

PART - C (1 x 15 Marks)

16. a) The turning moment diagram of a four-stroke engine is assumed to be represented by four triangles, the areas of which from the line of zero pressure are

Suction stroke = 440 mm^2

Compression stroke = 1600 mm^2

Expansion stroke = 7200 mm^2

Exhaust stroke = 660 mm^2

Each mm^2 of area represents 3 Nm of energy. If the resisting torque is uniform, determine the mass of the rim of a flywheel to keep the speed between 218 rpm and 222 rpm, when the mean radius of the rim is to be 1.25 m.

(OR)

b) A shaft 30 mm diameter and 1.5 m long has a mass of 16 kg per metre length. It is simply supported at the ends and carries three isolated loads 1 kN, 1.5 kN, and 2 kN at 0.4 m, 0.6 m and 0.8 m respectively from the left support. Find the frequency of the transverse vibrations.

i) Neglecting the mass of the shaft and

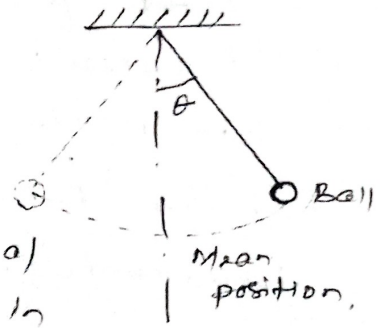
ii) Considering the mass of the shaft

Take the Young's modulus of the shaft material E as 200 GPa.

— VIBRATION —

Basic concept :-

The mass is inherent of body and elasticity causes relative motion among it's parts. When the body particles are displaced by the application of external force, the internal forces in the form of elastic energy are present in the body. These forces try to bring the body to it's original position. At equilibrium position, the whole of the elastic energy is converted into kinetic energy and body continues to move in opposite direction because of it. The whole of the kinetic energy is again converted into elastic energy due to which the body again returns to the equilibrium position. This way vibratory motion is repeated with exchange of energy. This phenomenon is called vibration.



— Swinging of simple pendulum shown in the fig. is an example of vibration.

Definitions :-

Periodic motion → a motion repeating itself after equal interval of time

Time period → time taken to complete one cycle.

frequency → No of cycles per unit time

mean displacement of vibrating

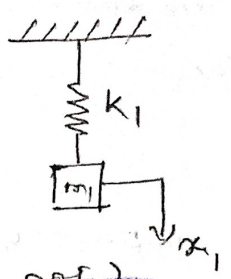
Natural frequency:- When no external force acts on the system after giving it an initial displacement the body vibrates. These vibrations are called free vibrations and their frequency is called natural frequency. It is expressed in rad/s or Hertz.

Fundamental mode of vibration:- The fundamental mode of vibration of a system is the mode having the lowest natural frequency.

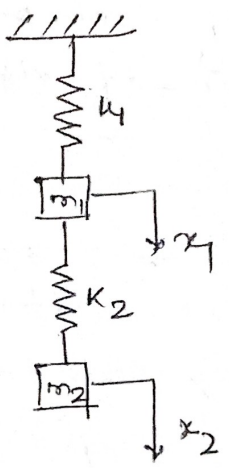
Degree of freedom:- The min^m no of independent coordinates required to specify the motion of a system at any instant is known as degrees of freedom of the system.

- In general it is equal to the no of independent displacements that are possible. This number varies from zero to infinity.

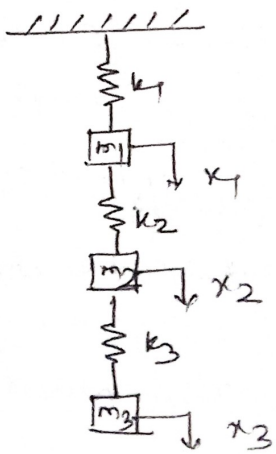
Examples of one, two and three degree of freedom system are shown in the figures.



(One DOF)



(Two DOF)



(Three DOF)

Simple Harmonic Motion :- The motion of a body to and fro about a fixed point is called Simple Harmonic motion. The motion is periodic and it is oscillatory in nature. It is directed towards the mean position and is proportional to its distance from mean position. The motion of a simple pendulum is an example of SHM.

Illustration

Damping :- It is the resistance due to motion of vibrating body. The vibration decreases due to resistance. So are called damped vibrations.

Resonance :- When the frequency of external periodic force equal to natural frequency of a vibrating body, the amplitude of vibration becomes maximum. This phenomenon is called resonance.

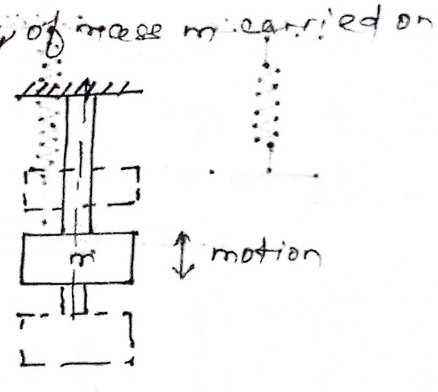
Parts of a Vibrating System :-

A simple vibrating system consists of three elements mass, spring and damper. The spring and damper form a vibrating body. Energy is exchanged from potential to kinetic energy. Kinetic energy is stored by mass in the form of KE ($\frac{1}{2}mv^2$), in the spring in the form of PE ($\frac{1}{2}kx^2$) and dissipated in the damper in the form of heat energy which

Types of vibrations:-

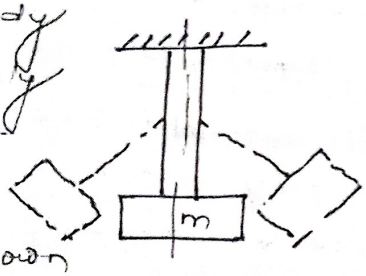
(i) Longitudinal vibrations:-

considering a case where a body of mass m carried on one end of a weightless spindle, the other end being fixed. If the mass 'm' moves up and down parallel to the spindle axis, it is said to execute longitudinal vibration.



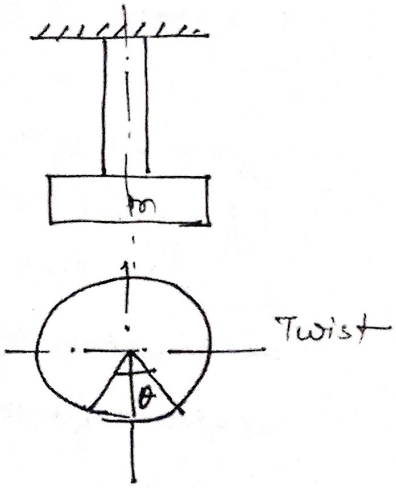
(ii) Transverse vibrations:-

When the particles of the body or shaft move approximately ⊥r to the axis of the shaft as shown in the figure, the vibration caused is known as transverse vibration.



(iii) Torsional vibrations:-

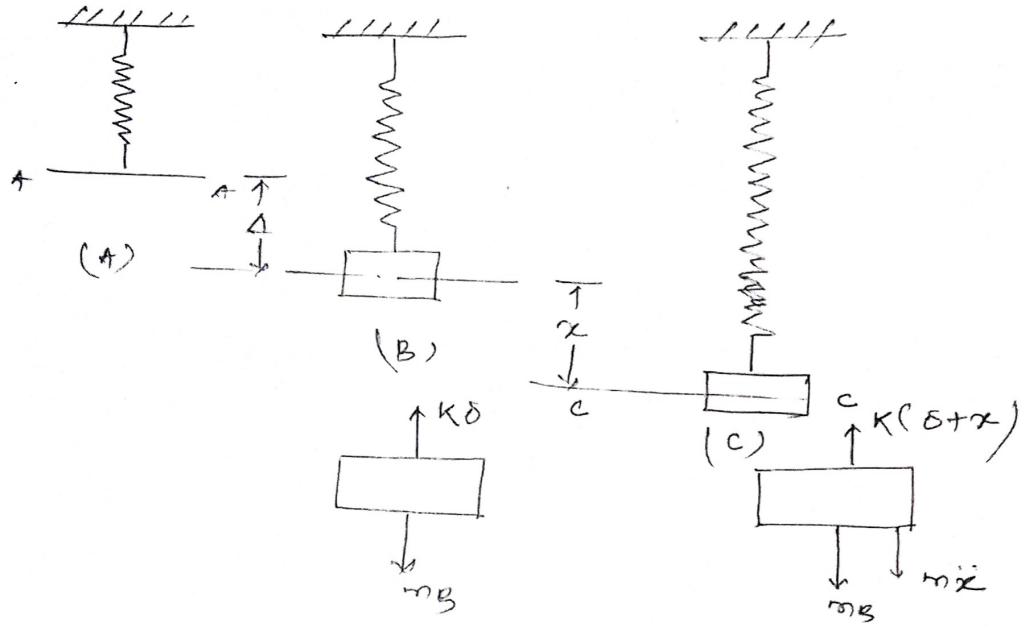
If the spindle get alternately twisted and untwisted on account of vibratory motion of the suspended disc, then it is undergoing torsional vibration



Transient vibrations:-

In ideal systems the free vibration continues indefinitely as there is no damping. The amplitude of vibration decays continuously because of damping (In practical system) and vanishes ultimately.

Calculation of Natural frequency of undamped Single DOF system



The natural frequency of a free undamped Single DOF system can be obtained from any of the following methods:

(i) Equilibrium method (D'Alembert's principle)

This method is based on the principle that ~~were~~ whenever a vibrating system is in equilibrium, the algebric sum of force and moments acting on it is zero. This is in accordance with D'Alembert's principle that the sum of external forces and inertia forces acting on a body in equilibrium, must be zero.

- Figure (A) shows a helical spring suspended vertically from a rigid support with its free end at A-A. If a mass is suspended from the free end the spring is stretched by a distance Δ and B-B becomes the equilibrium position. So Δ is the static deflection of the spring under the wt. of mass 'm'.

In equilibrium position, the gravitational pull W is balanced by a spring force, such that

$$mg = W = K\delta \quad \text{--- (1)}$$

Where δ = static deflection of spring in m,
K = spring stiffness.

- When the mass is displaced from its equilibrium position by a distance x and released, so after t

Resoring force $= W - K(\delta + x)$

$$\Rightarrow m\ddot{x} = \cancel{W} - \cancel{K\delta} - Kx \quad (\because W = K\delta)$$

$$\Rightarrow m\ddot{x} = -Kx$$

$$\Rightarrow m\ddot{x} + Kx = 0 \quad \text{--- (2)}$$

where $\ddot{x}$ acceleration of mass m. It is recognised as equation of a SHM.

The solution to eq. (2) is

$$x = A\cos\omega_n t + B\sin\omega_n t \quad \text{--- (3)}$$

Where $A, B \rightarrow$ constants, which can be obtained by substituting initial conditions

$\omega_n \rightarrow$ circular frequency of motion

~~substituting equation (3) in equation (2) we~~

~~we have~~ We have

$$\dot{x} = -\omega_n A\sin\omega_n t + \omega_n B\cos\omega_n t$$

$$\ddot{x} = -\omega_n^2 A\cos\omega_n t - \omega_n^2 B\sin\omega_n t$$

$$= -\omega_n^2 (A\cos\omega_n t + B\sin\omega_n t) \quad \text{--- (4)}$$

substituting the value of x and $\ddot{x}$ in eq. (2),

$$-\omega_n^2 (A\cos\omega_n t + B\sin\omega_n t) + \frac{K}{m}(A\cos\omega_n t + B$$

Since $A\cos\omega_n t + B\sin\omega_n t \neq 0$

$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{K}{m}}} \text{ rad/s} \quad \text{--- (5)}$$

The frequency of vibration $f_n = \frac{\omega_n}{2\pi}$

$$\Rightarrow \boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}} \text{ Hz.} \quad \text{--- (6)}$$

Time period $\boxed{T = \frac{1}{f_n} = 2\pi\sqrt{\frac{m}{K}}}$ --- (7)

(ii) Energy method:-

According to conservation of energy law

$$T + V = \text{constant} \quad \text{--- (8)}$$

Where T = kinetic energy

V = potential energy

Differentiation of above equation w.r.t time will be zero.

$$\frac{d}{dT}(T+V) = 0$$

Now $T = \frac{1}{2} m\dot{x}^2$

$V = \frac{1}{2} K x^2$

So $\frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} k x^2\right) = 0$

$$\Rightarrow \frac{1}{2} m\, 2\dot{x}\ddot{x} + \frac{1}{2} K\, 2x \cdot \dot{x} = 0$$

$$\Rightarrow m\ddot{x} + Kx = 0$$

so circular frequency $\boxed{\omega_n = \sqrt{\frac{K}{m}}}$ --- (9)

Rayleigh's Method:-

In this method, it is assumed that max KE at the mean position is equal to the max potential energy at the extreme position. The motion is assumed to be SHM, then

$$x = A\sin\omega_n t \quad \text{--- (10)}$$

where x = displacement of the body from mean position after time t

A = max^m displacement from mean position to the extreme position.

Differentiating eq. (10)

$$\dot{x} = \omega_n A \cos \omega_n t$$

max^m velocity at mean position

$$\dot{x} = \omega_n A$$

so max^m KE at mean position $= \frac{1}{2} m \dot{x}^2$

$$= \frac{1}{2} m \omega_n^2 A^2 \quad \text{— (11)}$$

max^m potential energy at the extreme position

$$= \frac{1}{2} K A^2 \quad \text{— (12)}$$

Equating equation (11) and (12)

$$\frac{1}{2} m \omega_n^2 A^2 = \frac{1}{2} K A^2$$

$$\Rightarrow \omega_n^2 = \frac{K}{m}$$

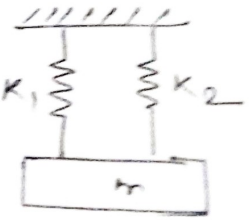
$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{K}{m}}} \quad \text{— (13)}$$

Equivalent spring stiffness for different combinations

Certain springs have more than one spring, which are joined either in series or in parallel. They can be replaced by a single spring of same stiffness.

(a) springs in parallel :-

The deflection of individual spring is equal to the deflection of the system.



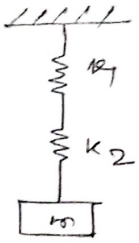
$$K_1 x + K_2 x = K_e x$$

$$\Rightarrow \boxed{K_e = K_1 + K_2}$$

where K_e is the equivalent spring stiffness.

(b) springs in series:-

Total deflection of the system is equal to the sum of deflection of individual springs.



$$x = x_1 + x_2$$

$$\Rightarrow \frac{Force}{k_e} = \frac{force}{k_1} + \frac{force}{k_2}$$

$$\Rightarrow \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \frac{k_1 + k_2}{k_1 k_2}$$

$$\Rightarrow \frac{1}{k_e} = \frac{k_1 + k_2}{k_1 k_2}$$

$$\Rightarrow \boxed{k_e = \frac{k_1 k_2}{k_1 + k_2}}$$

Natural frequency of spring

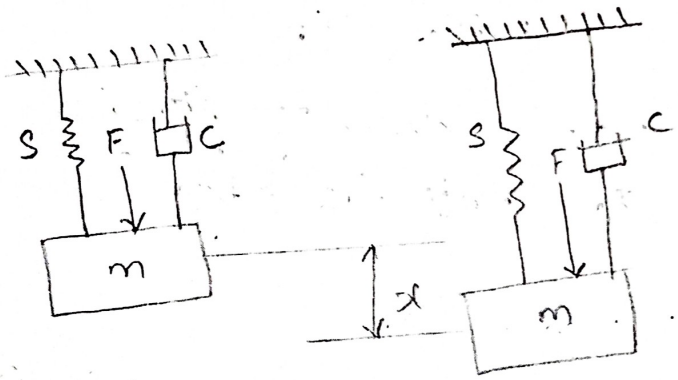
UNIT - 4

Forced Vibration

Definition

Defined as the vibration of a body, which is caused with the help of an external forces without removing it.

Frequency of damped forced vibration



$F = F_0 \sin\omega t$

Forces acting are

Inertia force $= \dfrac{m d^2x}{dt^2}$

Spring force $= Sx$

damping force $= C\dfrac{dx}{dt}$

Force applied, $F = F_0 \sin\omega t$

Applying D'Alembert's principle

Inertia force + Σ external forces $= 0$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + sx - F_0 \sin\omega t = 0$$

$\div$ by m

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{s}{m}x = \frac{F_0}{m}\sin\omega t \longrightarrow (1)$$

This is the characteristic equation or governing equation: damped forced vibration.

$$x = x_c + x_p$$

x_c - complementry function

x_p - particular integral.

$\underline{x_c = C.F}$

This is similar to the derivation of free damped vibration

considering practical damping (i.e) underdamping

$$x_c = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)$$

$\underline{x_p}$

apply $\frac{d}{dt} = D$ in (1)

$$D^2x + \frac{c}{m}Dx + \frac{s}{m}x = \frac{F_0}{m}$$

Assume $\frac{c}{m} = a$; $\frac{s}{m} = b$, $\frac{F_0}{m} = d$

$$(D^2 + aD + b)x = d\sin\omega t$$

$$x_p = \frac{d\sin\omega t}{D^2 + aD + b}$$

$D^2 = -\omega^2$

$$= \frac{d\sin\omega t}{-\omega^2 + aD + b}$$

$$= \frac{d\sin\omega t}{(b-\omega^2) + aD}$$

$\times$ and $\div$ by $(b-\omega^2) - aD$

$$x_p = \frac{d\sin\omega t \cdot [(b-\omega^2) - aD]}{((b-\omega^2) + aD)((b-\omega^2) - aD)}$$

$$x_p = \frac{d[(b-\omega^2)\sin\omega t - aD\sin\omega t]}{(b-\omega^2)^2 - a^2b^2}$$

$$= \frac{d[(b-\omega^2)\sin\omega t - a\omega\cos\omega t]}{(b-\omega^2)^2 + a\omega^2}$$

Assume $(b-\omega^2) = A\cos\phi$; $a\omega = A\sin\phi$

$$(b-\omega^2)^2 + a\omega^2 = A^2\cos^2\phi + A^2\sin^2\phi$$

$$[\cos^2\phi + \sin^2\phi = 1]$$

$$A = \sqrt{(b-\omega^2)^2 + (a\omega)^2}$$

$$x_p = \frac{d[(A\cos\phi)\sin\omega t - A\sin\phi\cos\omega t]}{A^2}$$

$$x_p = \frac{dA\,[\sin\omega t\cos\phi - \cos\omega t\sin\phi]}{A^2}$$

$[\sin A\cos B - \cos A\sin B = \sin(A-B)]$

$$= \frac{d(\sin(\omega t-\phi))}{A}$$

$$x_p = \frac{d(\sin(\omega t-\phi))}{\sqrt{(b-\omega^2)^2+(a\omega)^2}}$$

20/9/

Reapplying the values of a, b and d

$$x_p = \frac{\frac{F_0}{m}\sin(\omega t-\phi)}{\sqrt{\left(\frac{s}{m}-\omega^2\right)^2+\left(\frac{c}{m}\omega\right)^2}}$$

$$x_p = \frac{F_0\sin(\omega t-\phi)}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}}$$

$$x = x_c + x_p$$

$$x = Xe^{-\zeta\omega_n t}\sin(\omega_d t+\phi) + \frac{F_0\sin(\omega t-\phi)}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}}$$

Mostly x_c becomes negligible as compared to applied force $F = F_0\sin\omega t$

$$x = \frac{F_0\sin(\omega t+\phi)}{\sqrt{(s-m\omega^2)^2}}$$

Maximum displacement $x_{max} = \dfrac{F_0}{\sqrt{(S - m\omega^2)^2 + (C\omega)^2}}$

$[\because \sin(\omega t - \phi) = 1]$

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Frequency Ratio (r)

We know that maximum amplitude of forced vibration,

$$X_{max} = \frac{F_0}{\sqrt{(S - m\omega^2)^2 + (C\omega)^2}}$$

$$= \frac{F_0/S}{\sqrt{\left(1 - \frac{m}{S}\omega^2\right)^2 + \left(\frac{C\omega}{S}\right)^2}}$$

Now,

$$\frac{C\omega}{S} = \frac{C}{C_c} \cdot \frac{C_c}{2m} \cdot \frac{2m}{S} \cdot \omega$$

$$= \zeta \cdot \omega_n \cdot \frac{2m}{S} \cdot \omega$$

$$= \zeta \cdot \omega_n \cdot \frac{2}{S/m} \cdot \omega$$

$$= \zeta \cdot \omega_n \cdot \frac{2}{\omega_n^2} \cdot \omega$$

$$= \zeta \cdot 2 \cdot \omega/\omega_n$$

$$\boxed{\frac{C\omega}{S} = \zeta\, 2\omega/\omega_n}$$

$$x_{max} = \frac{F_0/S}{\sqrt{\left(1-\frac{\omega^2}{S/m}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{F_0/S}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

where r is the frequency ratio equal to $\frac{\omega}{\omega_n}$.

Phase lag (ϕ)

The phase angle is the ~~force~~ phase lag by which displacement vector lags the force vector.

$$x = X e^{-\zeta\omega_n t} \cdot \sin(\omega_d t + \phi_1)$$

$$+ \frac{F_0 \cdot \sin(\omega t - \phi)}{\sqrt{(S-m\omega^2)^2 + (c\omega)^2}}$$

we assumed that

$$A\cos\phi = b - \omega^2$$
$$A\sin\phi = a\omega$$

$$\tan\phi = \frac{a\omega}{b-\omega^2}$$

also

$$a = \frac{c}{m}, \quad b = \frac{s}{m}$$

$$\tan\phi = \frac{c/m\,\omega}{\frac{s}{m} - \omega^2}$$

$$= \frac{c\omega}{s - m\omega^2}$$

$\div$ by s

$$\tan\phi = \frac{c.\omega/s}{1 - \frac{m\omega^2}{s}} = \frac{2\zeta\omega_n}{1 - \frac{\omega^2}{s/m}}$$

$$= \frac{2\zeta\omega_n}{1 - \left(\frac{\omega}{\omega_n}\right)^2} = \frac{2\zeta\omega_n}{1 - r^2}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta\omega_n}{1 - r^2}\right)$$

Response to periodic forcing

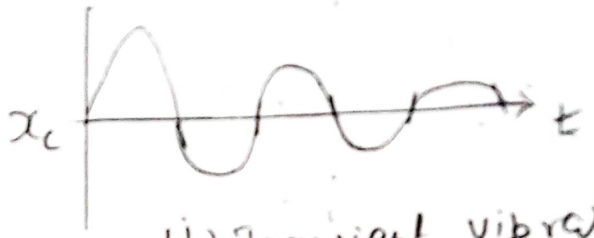
1. Transient vibration
2. Steady state vibration

$$x = X e^{-\zeta\omega_n t}\sin(\omega_d t + \phi_1) + \left[\frac{F_0 \sin(\omega t - \phi)}{\sqrt{(s - m\omega)^2 + (c\omega)^2}}\right]$$

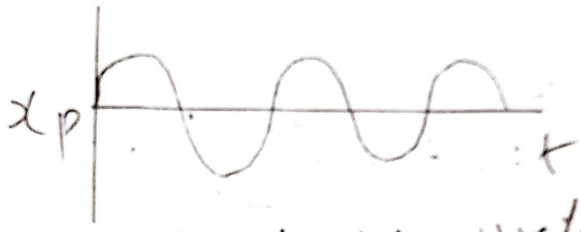
(1) (2)

First part (1) is called transient vibration

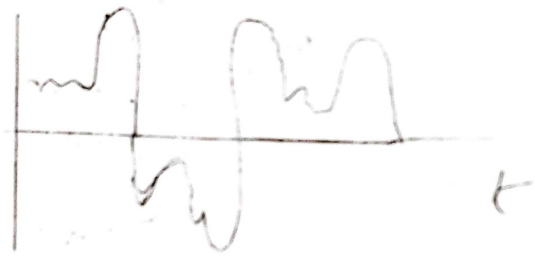
second part (2) is called as steady state vibration



(1) Transient vibration



Steady state vibration



A mass of 50 Kg is supported by an elastic structure of total stiffness 20 kN/m. Damping ratio of the system is 0.2. A simple harmonic disturbing force acts on the mass and at any time 't' seconds the force is 60 sin10t N. Find the amplitude of vibrations and phase angle caused by the damping

$m = 50\ Kg$

$S = 20\ KN/m$

$F_0 \sin\omega t = 60 \sin 10\, t$

$\zeta = 0.2$

$\because F_0 = 60 \quad \omega = 10$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{20 \times 10^3}{50}}$$

$$\omega_n = 20\ rad/s$$

$$r = \frac{\omega}{\omega_n}$$

$$r = \frac{10}{20} = 0.5$$

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$= \frac{60/20\times10^3}{\sqrt{(1-0.5^2)^2 + (2\times0.2\times0.5)^2}}$$

$$= \frac{3\times10^{-3}}{\sqrt{0.562 + 0.04}} = \frac{3\times10^{-3}}{\sqrt{0.602}}$$

$$= 3.86 \times 10^{-3}\ m$$

$$= 3.86\ mm$$

(ii) Phase angle caused by damping (ϕ)

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

$$= \tan^{-1}\left(\frac{2\times 0.2\times 0.5}{1-(0.5)^2}\right)$$

$$\theta = 14.93^\circ$$

A mass of 50 kg suspended from V end supported by an elastic structure of stiffness 20 kN/m. The simple harmonic disturbing force in N express as 75 cos 12 t acts on the mass. If t is expressed in seconds and the damping force is 0.25. Find the amplitude of vibration and the phase angle caused by the damping

$m = 50\,kg$

$S = 20\,kN/m$

$F = 75\cos 12t$

$\therefore F_0 = 75 \quad \omega = 12$

$\zeta = 0.25$

$$\omega_n = \sqrt{\frac{S}{m}}$$

$$= \sqrt{\frac{20\times 10^3}{50}}$$

$\omega_n = 20$ rad/s

$$r = \frac{\omega}{\omega_n} = \frac{12}{20} = 0.6$$

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$= \frac{75/20\times10^3}{\sqrt{(1-0.6^2)^2 + (2\times0.25\times0.6)^2}}$$

$$= \frac{3.75\times10^{-3}}{\sqrt{0.409 + 0.09}} = 5.3\times10^{-3}\ m$$

$\therefore x_{max} = 5.3$ mm

(ii) Phase angle caused by damping (ϕ)

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

$$= \tan^{-1}\left(\frac{2\times0.25\times0.6}{1-0.6^2}\right)$$

$$\phi = 25.11^\circ$$

Lograthmic Decrement

Formula $\delta = \frac{1}{n} \ln \left(\frac{x_0}{x_n} \right)$

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}$$

22/9/14 A mass of 10 kg is suspended from 1 end of helical spring and other end is being fixed. The stiffness of the spring is 10 N/mm. The viscos damping causes the amplitude to decrease to one tenth of the initial value in the 4 complete oscillations. If the periodic force of $150 \cos 50t$ N is applied at the mass in the vertical direction. Find the amplitude of force vibration. What is its value of resonance

Given data

$m = 10$ kg

$S = 10$ N/mm $= \frac{10}{10^{-3}}$ N/mm

$x_4 = \frac{1}{10} x_0 \Rightarrow \frac{x_0}{x_4} = 10$

$F = F_0 \cos 50t$

$F = 150 \cos 30t$

$F = F_0 \cos \omega t$

$F_0 = 150 \text{ N}$

$\omega = 30 \text{ rad/s}$

$$\delta = \frac{1}{n} \ln\left(\frac{x_0}{x_n}\right)$$

$n = 4$

$$\delta = \frac{1}{4} \ln\left(\frac{x_0}{x_4}\right)$$

$$= \frac{1}{4} \ln(10)$$

$$\delta = 0.5756$$

Also $\delta = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

$$0.5756 = \frac{2 \times \pi \times \zeta}{\sqrt{1-\zeta^2}}$$

$$\sqrt{1-\zeta^2} = 10.91\,\zeta$$

$$1-\zeta^2 = 119.03\,\zeta^2$$

$$\frac{1}{\zeta^2} - 1 = 119.03$$

$$\frac{1}{\zeta^2} = 120.03 \Rightarrow \zeta^2 = \frac{1}{120.03}$$

$\zeta = 0.0912$

(i) Amplitude of forced vibration (X_{max})

$$X_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$r = \frac{\omega}{\omega_n}$; $\omega_n = \sqrt{s/m} = \sqrt{\frac{10^4}{10}}$

$= 31.62$ rad/s

$r = \frac{50}{31.62} = 1.581$

$$X_{max} = \frac{150/10^4}{\sqrt{(1-1.581^2)^2 + (2 \times 0.0912 \times 1.581)^2}}$$

$= 9.822 \times 10^{-3}$ m

$X_{max} = 9.822$ mm

(ii) when resonance occurs

$\omega = \omega_n$

$r = \frac{\omega}{\omega_n} = 1$

$$X_{max} = \frac{F_0/s}{\sqrt{0 + (2\zeta \times 1)^2}}$$

$$= \frac{F_0/s}{\sqrt{(2\zeta)^2}} = \frac{F_0/s}{2\zeta}$$

$$= \frac{150/10^4}{2 \times 0.0912}$$

$$= 0.0822 \text{ m}$$

$$X_{max} = 82.236 \text{ mm}$$

Harmonic exciting force of 25 N is acting on a machine part. Which is having a mass of 2 Kg and is vibrating in a visious medium. This exiting force causes a resonant amplitude of 12.5 mm with a period of 0.2 second. Determine the damping co-efficient.

Given data

$$X_{max} = 12.5 \text{ mm}$$
$$= 0.0125 \text{ m}$$

$$F_0 = 25 \text{ N}$$

$$m = 2 \text{ kg}$$

$$t_p = 0.2 \text{ s}$$

To find $\xi = ?$

Solution

Find out the stiffness

$$\omega = \frac{2\pi}{t_p}$$

$$= \frac{2\pi}{0.2} = 31.4 \text{ rad/s}$$

$$= \frac{150/10^4}{2 \times 0.0912}$$

$$= 0.0822 \text{ m}$$

$$X_{max} = 82.236 \text{ mm}$$

Harmonic exciting force of 25 N is acting on a machine part. Which is having a mass of 2 Kg and is vibrating in a visious medium. This exiting force causes a resonant amplitude of 12.5 mm with a period of 0.2 second. Determine the damping co-efficient.

Given data

$$X_{max} = 12.5 \text{ mm}$$
$$= 0.0125 \text{ m}$$

$$F_0 = 25 \text{ N}$$

$$m = 2 \text{ Kg}$$

$$t_p = 0.2 \text{ s}$$

To find

$$\xi = ?$$

Solution

Find out the stiffness

$$\omega = \frac{2\pi}{t_p}$$

$$= \frac{2\pi}{0.2} = 31.4 \text{ rad/s}$$

As resonant amplitude occurs here

$\omega = \omega_n$

$\omega = \omega_n = 31.416$ rad/s

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{s}{2}} = 31.416$$

$s = 1973.92$ N/m

Also at resonance $\omega = \omega_n$

$$X_{max} = \frac{F_0/s}{2\zeta}$$

$$12.5\times10^{-3} = \frac{25/1973.92\times10^{-3}}{2\times\zeta}$$

$$\zeta = 5.06\times10^{-4}\times10^{-3}$$

$$\zeta = 0.506$$

$$\therefore c = 2m\omega_n\zeta$$

$$= 2\times2\times31.416\times0.5066$$

$$c = 63.66 \text{ N/m/s}$$

In the above problem if the system is excited by harmonic force of frequency

$$\omega = \frac{2\pi}{t_p}$$

$$\omega = 2\pi f \qquad \left[\because f = \frac{1}{t_p}\right]$$

$$\omega = 2\pi \times 4$$

$$\omega = 25.12 \text{ rad/s}$$

From previous problem

$$\omega_n = 31.416 \text{ rad/s}$$

$$X_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$r = \frac{\omega}{\omega_n} = \frac{25.12}{31.416} = 0.799 \Rightarrow 0.8$$

$$= \frac{25/1973.92}{\sqrt{(1-0.8^2)^2 + (2 \times 0.5066 \times 0.8)^2}}$$

$$= \frac{0.012}{\sqrt{0.129 + 0.657}}$$

$$X_{max} = 13.53 \text{ mm}$$

If $\xi = 0$ removing damping

$$X_{max} = \frac{23 / 1973.92}{\sqrt{(1-0.8^2)^2 + (2 \times 0 \times 0.8)^2}}$$

$$= \frac{0.012}{\sqrt{0.129}}$$

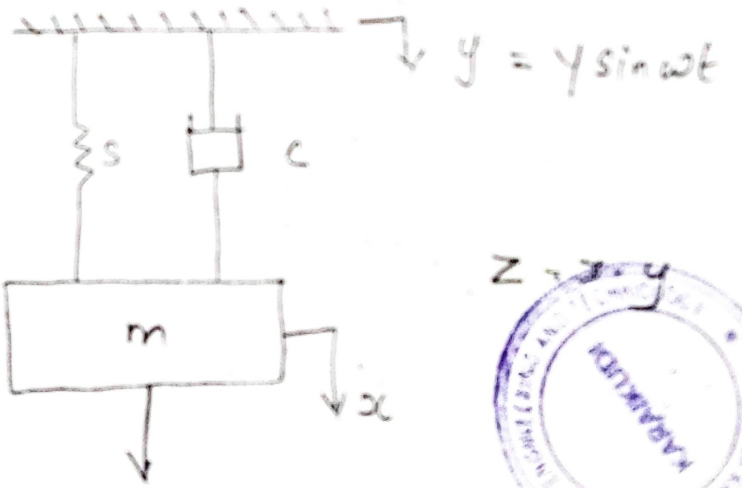
$$X_{max} = 33.41 \text{ mm}$$

$\therefore$ Increase in amplitude

$$= 33.41 - 13.53$$

$$= 19.88 \text{ mm}$$

Forced vibration due to excitation of support [support motion



According to D'Alembert's principle

Inertia force + Σ external forces = 0

$$m \cdot \frac{d^2x}{dt^2} + s(x-y) + c\left(\frac{dx}{dt} - \frac{dy}{dt}\right) = 0$$

$$m \cdot \frac{d^2x}{dt^2} + sx - sy + c\frac{dx}{dt} - c\frac{dy}{dt} = 0$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + sx = c\frac{dy}{dt} + sy \longrightarrow (1)$$

Apply $y = Y \sin\omega t$

$$m \cdot \frac{d^2x}{dt^2} + c\frac{dx}{dt} + sx = s(y \sin\omega t) + cy\omega\cos\omega t$$

$$m \cdot \frac{d^2x}{dt^2} + c\frac{dx}{dt} + sx = y\left[s \cdot \sin\omega t + c\omega\cos\omega t\right]$$

Now assume $s = A\cos\alpha$ $\quad c\omega = A\sin\alpha$

$$A^2(\sin^2\alpha + \cos^2\alpha) = s^2 + (s\omega)^2$$

$$A = \sqrt{s^2 + (s\omega)^2}$$

$$\tan\alpha = \frac{A\sin\alpha}{A\cos\alpha} = \frac{c\omega}{s}$$

$$\alpha = \tan^{-1}\left[\frac{c\omega}{s}\right]$$

$$= \tan^{-1}(2\zeta\omega/\omega_n)$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + sx = Y\left[A\cos\alpha \sin\omega t + A\sin\alpha\cos\omega t\right]$$

$$= AY\left[\sin\omega t\cos\alpha + \cos\omega t\cos\alpha \sin\alpha\cos\beta + \cos\alpha\sin\beta\right]$$

$$= AY\left[\sin(\omega t + \alpha)\right] \quad \text{Sin of angle}$$

$\rightarrow$ (2)

$\div$ by m

$$\frac{d^2x}{dt^2} + \frac{s}{m}\frac{dx}{dt} + \frac{s}{m}x = \frac{AY}{m}\left[\sin(\omega t + \alpha)\right]$$

W.K.T

Equation of motion for forced vibration with force applied on mass [Harmonic forcing]

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{s}{m}x = \frac{F_0}{m}\sin\omega t$$

$\rightarrow$ (3)

compare eqn (2) & (3) out of which, the solution (2) is known already

For (3)

$$x_{max} = \frac{F_0}{\sqrt{(s - m\omega^2)^2 + (c\omega)^2}}$$

For (2)

$$x_{max} = \frac{AY}{\sqrt{(s - m\omega^2)^2 + (c\omega)^2}}$$

$$x_{max} = \frac{Y\sqrt{s^2+(c\omega)^2}}{\sqrt{(s^2-m\omega^2)^2+(c\omega)^2}}$$

$$\frac{x_{max}}{Y} = \frac{\sqrt{s^2+(c\omega)^2}}{\sqrt{(s^2-m\omega^2)^2+(c\omega)^2}}$$

$\div$ by s

$$\frac{x_{max}}{Y} = \frac{\frac{1}{s}\sqrt{s^2+(c\omega)^2}}{\frac{1}{s}\sqrt{(s^2-m\omega^2)^2+(c\omega)^2}}$$

$$\frac{x_{max}}{Y} = \frac{\sqrt{1+\left(\frac{c\omega}{s}\right)^2}}{\sqrt{\left(1-\frac{m\omega^2}{s}\right)^2+\left(\frac{c\omega}{s}\right)^2}}$$

$$= \frac{\sqrt{1+\left(2\zeta\omega/\omega_n\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\omega/\omega_n\right)^2}}$$

$$\frac{x_{max}}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

where $r = \frac{\omega}{\omega_n}$

Introduction:-

speed variation in an engine occurs in two ways:

- cyclic variation
- variation of speed over a number of revolutions

Cyclic variations:-

Cyclic variation occurs because of variation in the turning moment of the engine. These variations can be reduced by mounting a suitable flywheel on the shaft.

Variation of speed over a number of revolutions:-

Variation of speed over a number of revolutions is because of variation of load on the engine.

- In this case a governor is mounted which controls the mean speed of the engine by regulating fuel supply to it.
- When the load increases, speed decreases and it's necessary to increase the fuel supply by opening the throttle valve to maintain mean speed of the engine and vice versa.

Difference betn Governor and Flywheel:-

Governor	Flywheel
1. Maintains the variation of mean speed within prescribed limit	1. Controls the speed variations in engine caused due to fluctuations of turning moment.
2. It regulates the speed over a period of time.	2. It regulates the speed during a cycle only.

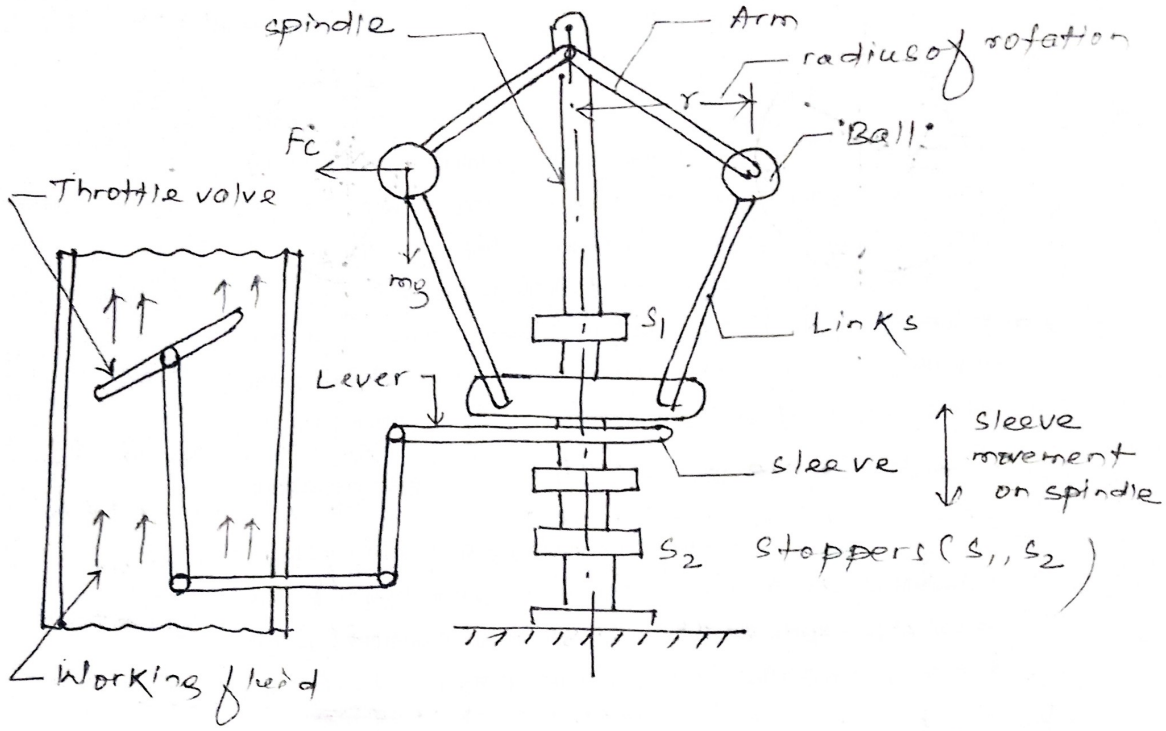
Governor	Flywheel
charge of the engine or prime mover	It stores energy and give it up whenever required in a cycle.

Types of Governor:-

- Governor
  - Centrifugal governor
    - Pendulum type
      - Watt Governor
    - Loaded governor.
      - Dead wt.
        - Porter governor
        - Proell governor
      - Spring controlled
        - Hartnell governor
        - Wilson Hartnell governor
        - Hartung governor
        - Pickering governor.
  - Inertia governor.

classification of different governors are explained in the above figure.

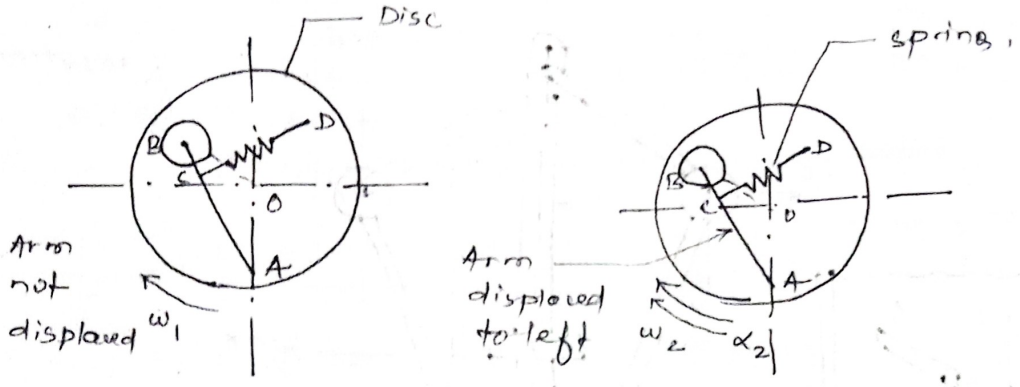
Centrifugal Governor:-



Centrifugal governor consists of two balls connected to spindle through arms. The upper arms are keyed to the spindle and lower arms (links) are connected to the sleeve. The sleeve is free to slide on the spindle. The balls rotate with spindle (shaft), giving rise to the centrifugal force which radially ~~acts~~ acts outwards. When the speed increases, the balls rotate at a larger radius and the sleeve slides upwards on the spindle, ~~this~~ and with the ~~two~~ lever the throttle is closed to the required extent.

- With the decrease in speed the governor balls rotate at smaller radius of rotation, compelling the sleeve to move down on the spindle. The downward movement of sleeve opens the throttle to the required output to admit the required fuel into

Inertia Governor:-



- The balls ~~are~~ is attached to arm AB and CD is the spring which controls the displacement of governor and changes amount of fuel to be supplied to the engine to meet the variations.

- When the load on engine ~~decreases~~ increases, the speed of the disc increases to ω_2 and is subjected to an angular acceleration α_2 also.

  and $\boxed{\omega_2 = \omega_1 + \alpha_2 t}$

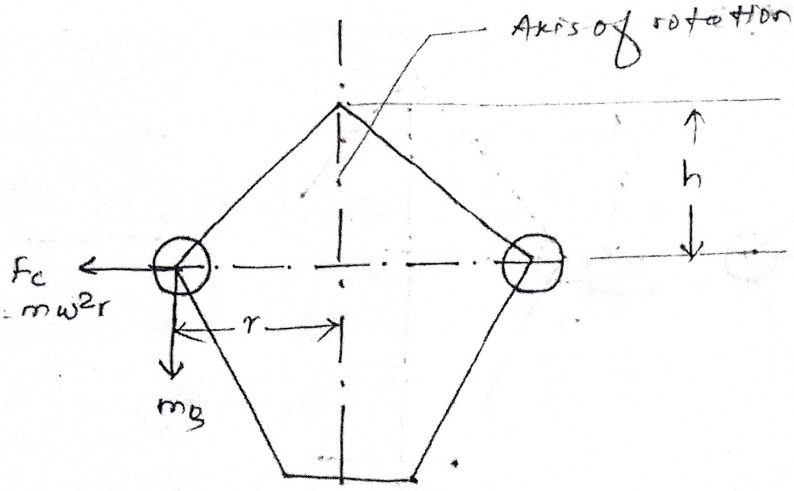
  The arm is displaced to left due to centrifugal force on the governor ball and the energy supplied to the engine is cut-off till new equilibrium position is gained.

Terminology:-

Following terms are used in governor:-

(a) Height of governor:- vertical distance from centre of ball to point on the spindle axis where the axes of arms intersect.

- It is denoted by 'h'.



(b) centrifugal force :-

centrifugal force $\boxed{F_c = mr\omega^2}$

Where m = mass of the ball in Kg.

r = radius of rotation in m.

ω = angular speed rad/s

(c) controlling force :-

It is an equal and opposite force to that of the centrifugal force.

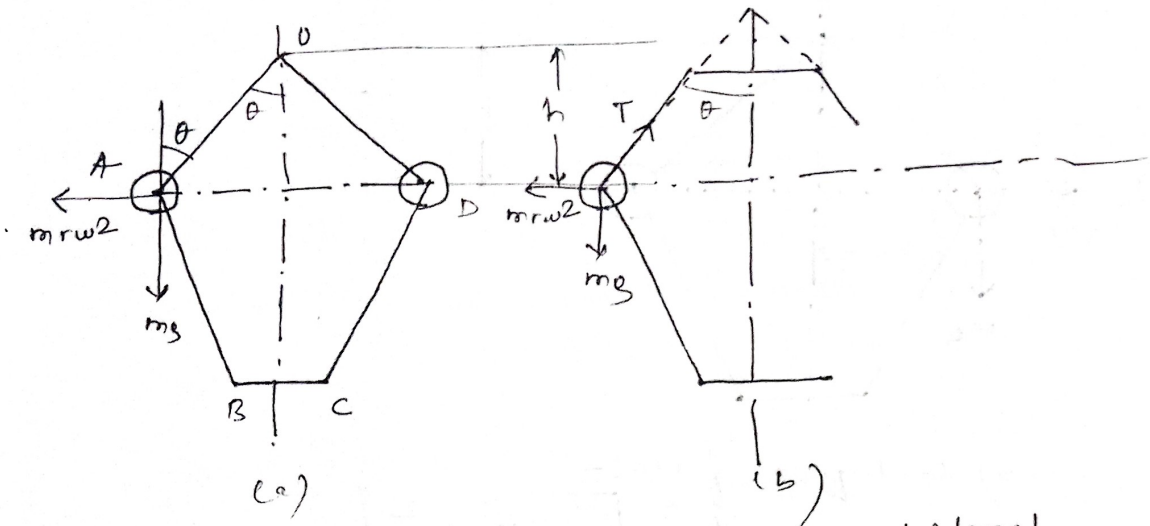
(d) Equilibrium speed :- speed at which governor balls, arms, sleeve etc are in equilibrium and there is no upward or downward movement of sleeve.

(e) Radius of rotation :- It is the horizontal distance between centre of ball and the axis of rotation, denoted by 'r'

(f) Mean equilibrium speed :- It is the speed at the mean position of the ball & sleeve.

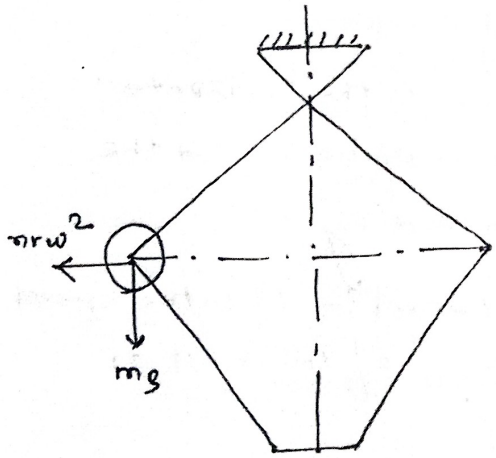
(g) Sleeve lift :- It is the vertical distance travelled by sleeve on the spindle in equilibrium speed.

WATT GOVERNOR :-



Watt governor is the simplest form of centrifugal governor. This governor is named after Watt who used it for steam engines.

- It is basically of three types depending upon the position of upper arms.
- When arms intersect at spindle axis, it is known as pinned arm type watt governor. (fig-a)
- The open-arm and cross-arm type watt governors are shown in fig. (b) and (c)



(c)

- In each of these three cases the lower arms i.e the links are fixed to the sleeve.
- When speed increases the balls moves outwards due to centrifugal force and the sleeve upwards on the spindle through the links and the vice versa.

Let m = mass of each ball in Kg.

ω = angular velocity of the balls about the spindle axis in rad/s.

The force of friction always acts in a direction opposite to that of the motion. When the sleeve moves up, the force of friction acts in downward direction and total downward force acting on the sleeve is $(Mg+f)$. Similarly when the sleeve moves down, the force on the sleeve will be $(Mg-f)$. In general the net force acting on sleeve is $(Mg \pm f)$

Let h = ht. of the governor

r = distance of center of each ball from the spindle axis.

- Now from the geometry $\triangle BAO$ is a kinematically equivalent to a slider crank mechanism with B as slider (vertical motion), the instantaneous centre of rotation of the link AB is at I for the given configuration of the governor.

- considering equilibrium of left hand half of the governor and taking moment about I

$$\Sigma M_I = 0$$

$$mr\omega^2 \times a = mg \times c + \frac{Mg \pm f}{2}(c+b)$$

$$\text{or } mr\omega^2 = mg \times \frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a}+\frac{b}{a}\right)$$

$$= mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

where $K = \frac{\tan\beta}{\tan\theta}$

From the above equation

$$mr\omega^2 = \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

$$\Rightarrow \boxed{h = \frac{g}{\omega^2} + \frac{(Mg \pm f)(1+K)}{m\omega^2}}$$

Q.1 A Watt governor runs at 100 rpm. Determine the height of the governor. If the speed of governor increases to 102 rpm find the change in vertical height.

Given: $N_1 = 100$ rpm $\quad N_2 = 102$ rpm.

initial height $h_1 = \frac{895}{N_1^2} = \frac{895}{(100)^2} = 0.0895$ m

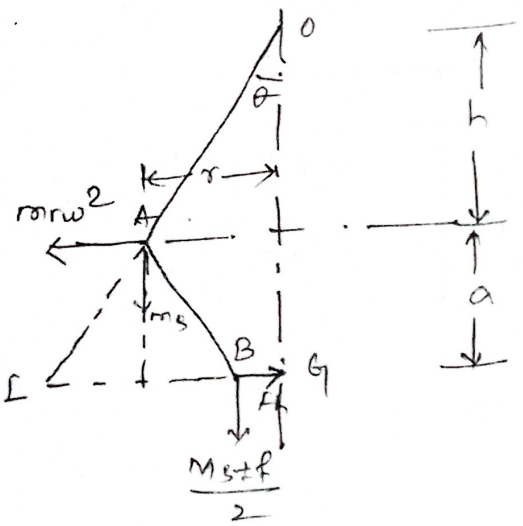
final height $h_2 = \frac{895}{(102)^2} = 0.086$ m

change in vertical height $= h_1 - h_2$

$= 0.0895 - 0.0860$

$= 0.0035$ m $= \boxed{3.5 \text{ mm}}$

Q.2 In a porter governor, each arm is 400 mm long. The lower arms are attached to the sleeve at a distance 45 mm from the axis. Each ball has a mass of 8 kg and the load on the sleeve is 60 kg. What will be the equilibrium speeds for two extreme radii of 250 mm and 300 mm of rotation of governor.



Given data:-

$m = 8$ kg $\quad M = 60$ kg.

$BG = 45$ mm $\quad OA = 400$ mm

Now we have

$m r \omega^2 = \tan\theta \left[mg + \frac{Mg}{2}(1+k) \right]$

(k = for coef friction neglected)

(i) when $r = 250$ mm

$\tan\theta = \frac{r}{h} = \frac{250}{\sqrt{400^2 - 250^2}} = 0.8$

$h = (250-45)/4$

Now $a = \sqrt{(AB)^2 - b^2}$

$= \sqrt{(400)^2 - (205)^2} = 343.4$ mm

So $K = \frac{b/a}{0.8} = \frac{205/343.4}{0.8} = \boxed{0.746}$

So we have

$8 \times 0.25 \times \omega^2 = 0.8\left[8 \times 9.81 + \frac{60 \times 9.81}{2}(1 + 0.746)\right]$

$\Rightarrow 2\omega^2 = 0.8(78.48 + 513.85)$

$\Rightarrow \omega^2 = 237$

or $\omega = 15.39$

Now $\omega = \frac{2\pi N}{60} = 15.39$

$\Rightarrow N = \frac{15.39 \times 60}{2\pi} = \boxed{147 \text{ rpm}}$

(ii) when radius $r = 300$ mm

$\tan\theta = \frac{r}{h} = \frac{300}{\sqrt{400^2 - 300^2}} = 1.134$

$b = 300 - 45 = 255$ mm

$a = \sqrt{400^2 - 255^2} = 308.2$ mm

$K = \frac{\tan\beta}{\tan\theta} = \frac{b/a}{1.134} = \frac{(255/308.2)}{1.134} = 0.73$

So,

$8 \times 0.3 \times \omega^2 = 1.134\left[8 \times 9.81 + \frac{60 \times 9.81}{2}(1 + 0.73)\right]$

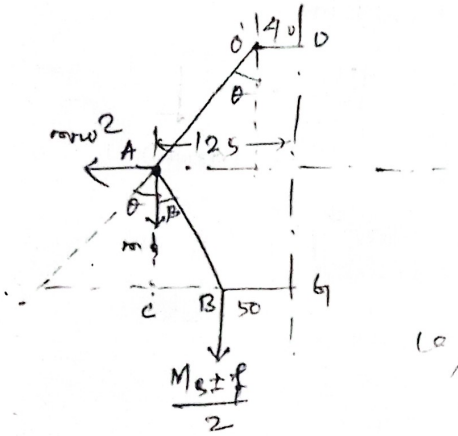
$\Rightarrow 2.4\omega^2 = 1.134(78.48 + 509.139)$

$\Rightarrow \omega^2 = 277.6$

$\Rightarrow \omega = 16.66 = \frac{2\pi N}{60}$

$\Rightarrow \boxed{N = 159.1 \text{ rpm}}$

Q.2 Each arm of a porter governor is 250 mm long. The upper ~~arm~~ and lower arm are pivoted to links of 40 mm and 50 mm respectively from axis of rotation. Each ball has a mass of 5 Kg and sleeve mass is 50 Kg. The force of friction on sleeve mechanism is 40 N. Determine the range of speed of the governor for extreme radii of rotation of 125 mm and 150 mm.



Given data:-

arm length = 250 mm

OO' = 40 mm, BG = 50 mm

m = 5 Kg, M = 50 Kg

f = 40 N, r_1 = 125 mm

r_2 = 150 mm.

(a) when r_1 = 125 mm

$$\tan\theta = \frac{(125-40)}{\sqrt{250^2-85^2}} = \frac{85}{235.1} = .361$$

$$\Rightarrow \boxed{\theta = 19.87^\circ}$$

$$\tan\beta = \frac{125-50}{\sqrt{250^2-75^2}} = \frac{75}{238.48} \Rightarrow \boxed{\beta = 17.47^\circ}$$

$$K = \tan\beta / \tan\theta = \boxed{.872} \Rightarrow \tan\beta = .215$$

Using the relation

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg-f}{2}(1+K)\right]$$ (∵ as the sleeve moves down, force of friction act upwards)

$$\Rightarrow 5\times 0.125\times\omega^2 = 0.361\left[5\times 9.81 + \frac{50\times 9.81-40}{2}(1+.872)\right]$$

$$\Rightarrow 0.625\omega^2 = 169.929$$

$$\Rightarrow \omega = 16.489$$

$$\Rightarrow N = \frac{16.489\times 60}{2\pi} = \boxed{157.45 \text{ rpm}}$$

(b) when r = 150 mm

150-40 110 0.49

$$\tan\beta = \frac{\quad}{\sqrt{250^2 - 100^2}} = \frac{\quad}{229.128} = 0.44$$

$$K = \frac{\tan\beta}{\tan\theta} = \frac{0.44}{0.49} = \boxed{0.897}$$

Now, we have

$$mr\omega^2 = \tan\theta \left[ms + \frac{Ms + f}{2}(1+K) \right]$$

$$\Rightarrow 5 \times 0.15 \times \omega^2 = 0.49 \left[5 \times 9.81 + \frac{50 \times 9.81 + 40}{2}(1 + 0.897) \right]$$

$$\Rightarrow 0.75\omega_2^2 = 270.592$$

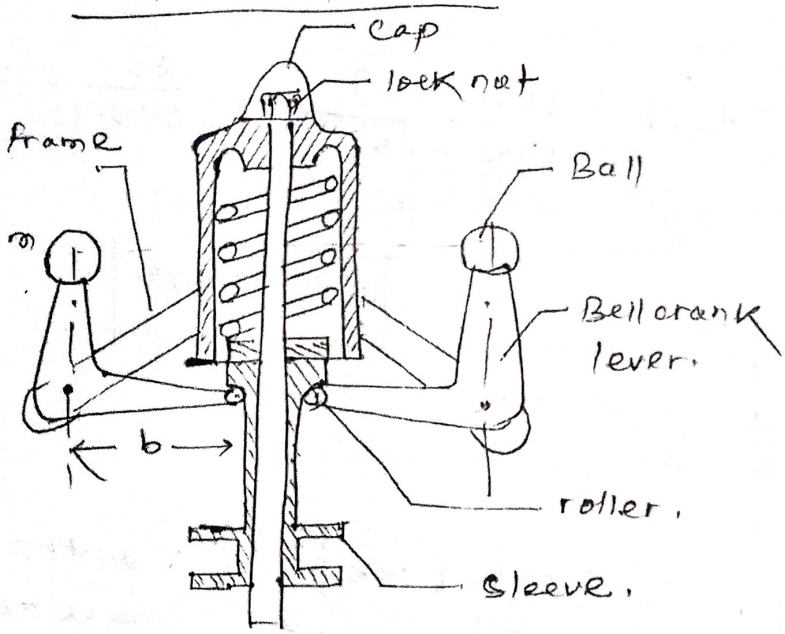
$$\Rightarrow \omega_2 = 18.99 \simeq 19$$

$$\Rightarrow N_2 = 181.38 \text{ rpm.}$$

Range of speed $= N_2 - N_1 = 23.93 \simeq \boxed{24 \text{ rpm}}$

(Ans)

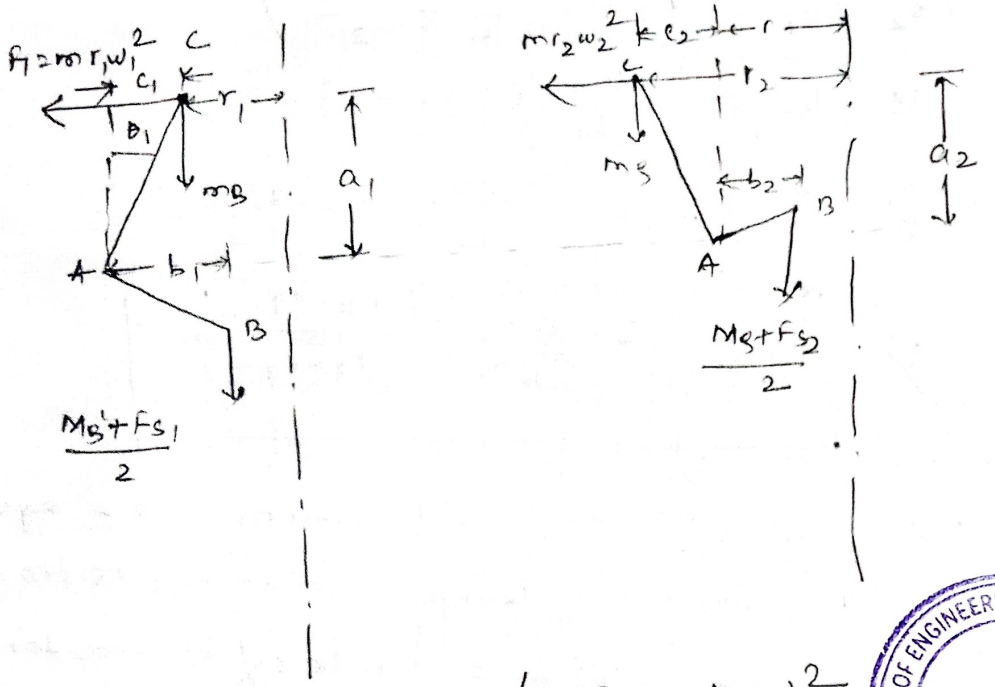
Hartnell Governor:-



Hartnell governor is as shown in the figure. The fra is keyed to the spindle and rotates with it. A compr spring is placed on the sleeve so that it can exert o sleeve. Two bell crank levers, each carrying a ball at one end and a roller at the other end are

the spindle upward against the spring force. If the force decreases, the sleeve moves downward. The spring force is adjusted with the help of locknut. The movement of sleeve is communicated to the throttle to perform necessary task.

The ~~three~~ positions of bell crank levers are shown in the figure.



Let F = centrifugal force $= mr\omega^2$

F_s = spring force,

Now taking moment about fulcrum A, $\Sigma M_A = 0$

$$F_1 a_1 = mg\, c_1 + \left(\frac{Mg + Fs_1}{2}\right) b_1$$

$$F_2 a_2 = mg\, c_2 + \left(\frac{Mg + Fs_2}{2}\right) b_2 \quad \text{—(i)}$$

[Neglecting friction force f]

In the working range of governors, θ is very small and so the obliquity effect may be neglected and we have

$$a_1 = a_2 = a, \quad b_1 = b_2 = b \quad c_1 = c_2 = 0.$$

So, $\Sigma \ldots \quad Mg + Fs_1 . h \quad$ —(ii)

substracting equation (iii) from (iii)

$$(F_2 - F_1)\, a = \left(\frac{F_{s_2} - F_{s_1}}{2}\right) b.$$

or $$\boxed{F_{s_2} - F_{s_1} = \frac{2a}{b} \cdot (F_2 - F_1)}$$

Now let s = stiffness of spring

h_1 = movement of sleeve.

$$F_{s_2} - F_{s_1} = h_1 s = \frac{2a}{b}\left(\frac{F_2 - F_1}{a}\right)$$

$$(\theta \times b)\, s = \frac{2a}{b}(F_2 - F_1)$$

$$\left(\frac{r_2 - r_1}{a}\right) \cdot bs = \frac{2a}{b}(F_2 - F_1)$$

$$\Rightarrow \boxed{s = 2 \cdot \frac{a^2}{b^2}\left(\frac{F_2 - F_1}{r_2 - r_1}\right)}$$

Q. The arms of a Hartnell governor are equal length. When the sleeve is in mid position, the masses rotate a circle of diameter 150 mm. Neglecting friction the equilibrium speed for this position is 360 rpm. Max^m variation of speed taking into account friction, is ±6% of mid-position speed for a max^m sleeve moment 30 mm. sleeve mass is 5kg and friction at the sleeve is 35N

– Assuming the power of governor is sufficient to overcome the friction, find 1% of change of speed on each side of mid-position find

(i) speed rotating mass of ball (ii) spring

21/02/2015

Q.3 For a Hartnell governor, following data are provided:

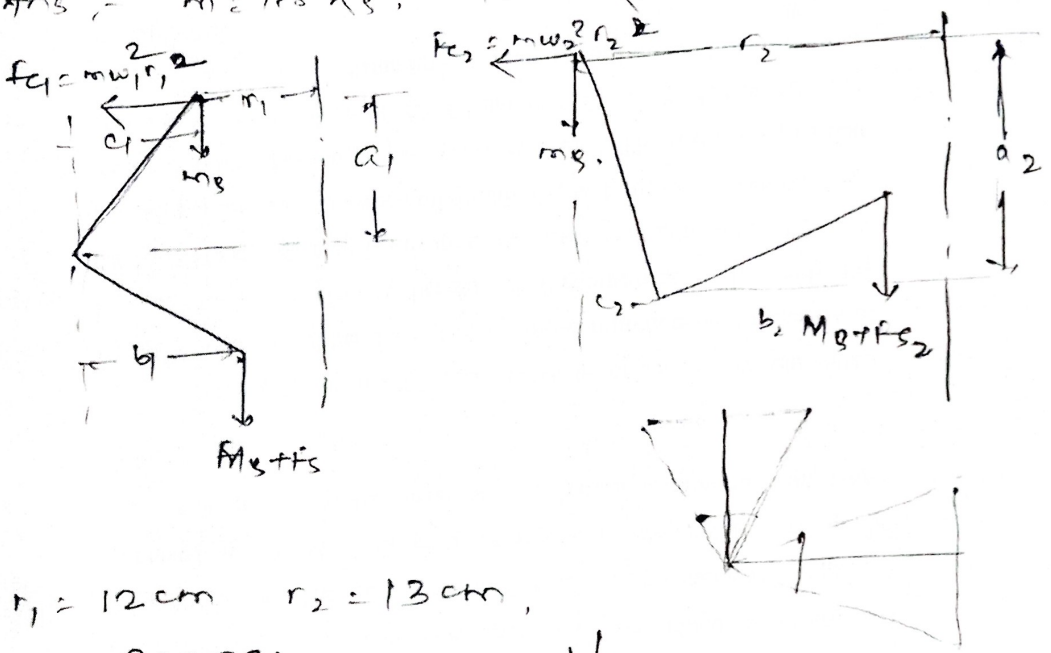
mass of each ball = 1.5 Kg., M = 50 Kg.

length of vertical arm of ball crank lever = 8.75 cm.

length of other arm of lever = 10 cm.

Speed corresponding to radii of rotation s of 12 cm and 13 cm are 296 rpm and 304 rpm respectively. Determine spring stiffness.

Ans:- m = 1.5 Kg, M = 50 Kg.



$r_1 = 12$ cm $\quad r_2 = 13$ cm,

$$\omega_1 = \frac{2\pi \times 296}{60} = 30.99 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi \times 304}{60} = 31.83 \text{ rad/s.}$$

$$F_{c_1} = m\omega_1 r_1^2 = 1.5 \times 30.99^2 \times (.12)^2 = \cancel{0.2603}$$

$$= 207.44 \text{ N}$$

$$F_{c_2} = 1.5 \times 31.83^2 \times .13 = 237.07 \text{ N}$$

$$\text{Spring stiffness } S = 2 \cdot \frac{a^2}{b^2}\left(\frac{F_{c_2} - F_{c_1}}{r_2 - r_1}\right)$$

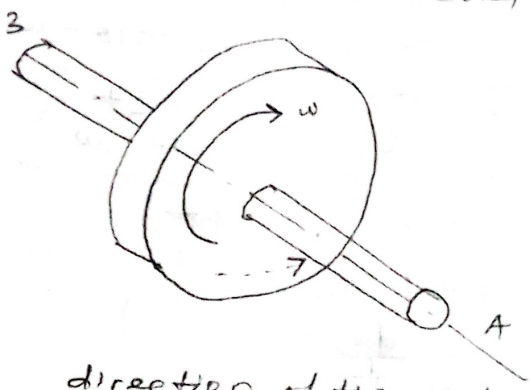
$$= 2 \times \frac{0.0875^2}{0.12} \times \left(\frac{237.07 - 207.44}{0.13 - 0.12}\right)$$

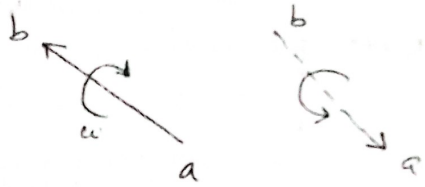
GYROSCOPE

Angular velocity :-

The angular velocity of a rotating body is specified by — magnitude of the velocity
— the direction of axis of rotation
— sense of rotation i.e. clockwise or anticlockwise.

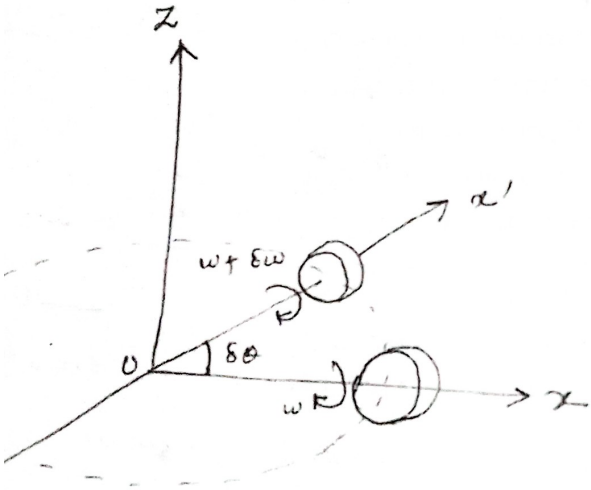
For example let's consider a rotor rotating in clockwise direction when viewed from A and. Its sense of

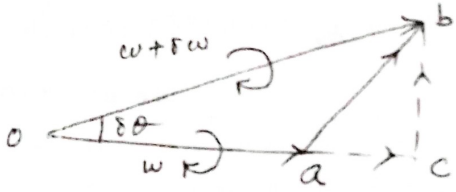




direction of the vector is from a to b is according to the screw rule.

Angular acceleration :-





Let's consider a rotor rotates about the horizontal axis ox at a speed of ω rad/s in a direction as shown in the figure. Let Oa represents its angular velocity.

Now, if the magnitude of the angular velocity changes to (ω+δω) and the direction of the axis of spin to ... in time δt), the vector Ob would represent

The change in angular velocity can be represented by joining ab. This vector ab can be resolved into two components

(i) component ac represents the ~~ang~~ change in angular velocity in a plane ⊥r to x axis

(ii) component cb represents the change in angular velocity in a plane ⊥r to y axis.

Now change in angular velocity ac

$$= (\omega + \delta\omega)\cos\delta\theta - \omega$$

Rate of change of angular velocity $= \dfrac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$

So angular acceleration $= \underset{\delta t \to 0}{Lt} \dfrac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$ $\cos\delta\theta \to 1$

So angular acceleration $= \dfrac{d\omega}{dt}$ —— (1)

Now, change of angular velocity cb

$$= (\omega + \delta\omega)\sin\delta\theta$$

Rate of change of angular velocity $\dfrac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$

So angular acceleration $= \underset{\delta t \to 0}{Lt} \dfrac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$, $\sin\delta\theta \to \delta\theta$

So, angular acceleration $= \dfrac{(\omega+\delta\omega)\,\delta\theta}{\delta t} = \omega \cdot \dfrac{d\theta}{dt}$ —— (2)

(∵ Neglecting small values)

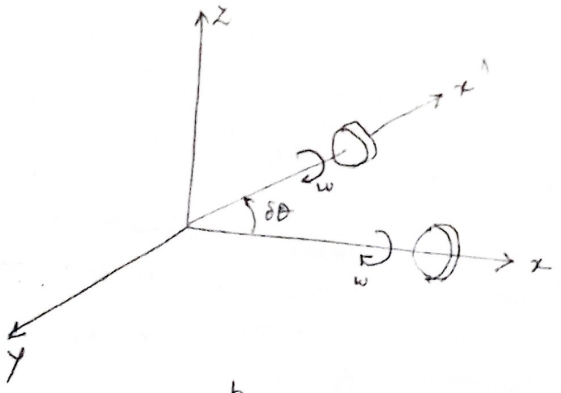
From equation (1) and (2), total angular acceleration

$$\boxed{\alpha = \frac{d\omega}{dt} + \omega\frac{d\theta}{dt}} \quad \text{—— (3)}$$

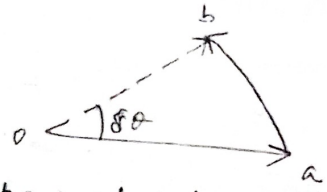
Equation (3) shows that, the total angular acceleration of the rotor is the sum of

(i) $\dfrac{d\omega}{dt}$, the change in angular velocity of rotor

Gyroscopic couple :-



Let I = momentum of inertia of a rotor and ω is it's angular velocity about horizontal axis of spin ox. Let's this axis of spin turn through a small angle δθ in the horizontal plane xy, to the position ox' in δt time.



The vectorial representation of the same is as shown. oa represents angular velocity vector when the axis is Ox and ob when the axis is changed to Ox'. Then ab represents the change in angular velocity due to change in direction of axis of spin.

- This change in angular velocity is clockwise when viewed from a towards b and is in the vertical plane xz. This change results in an angular acceleration

Now change in angular velocity $ab = \omega \times \delta\theta$

angular acceleration, $\alpha = \omega \frac{\delta\theta}{\delta t}$

Usually $\frac{\delta\theta}{\delta t}$, the angular velocity of axis of spin is called angular velocity of precession and is denoted by ω_p.

So angular acceleration $= \omega . \omega_p$

And ~~angular~~ torque required to produce this acceleration is called gyroscopic couple or torque.

Gyroscopic torque $= I\alpha$

$= \boxed{I \omega \omega_p}$

Ox → axis of spin
Oz → axis of precession
Oy → axis of gyroscopic couple.
yz → plane of spin
xy → plane of precession
xz → plane of gyroscopic couple

- The torque obtained from the above equation is required to cause the axis of spin to precess in the horizontal plane and is known as active gyroscopic torque.
- The effect of gyroscopic couple on a rotating body is known as the gyroscope effect on the body.

Q.1 A uniform disc having a mass of 8 kg and radius of gyration of 150 mm is mounted on one end of a horizontal arm of length 200 mm. The other end of the arm can rotate freely in a bearing. The disc is given a clockwise spin of 250 rpm when seen from the end. Determine the motion of disc if the arm remains horizontal.

$m = 8\ kg$, $l = 200\ mm = 0.2\ m$, $k = 0.15\ m$, $N = 240\ rpm$

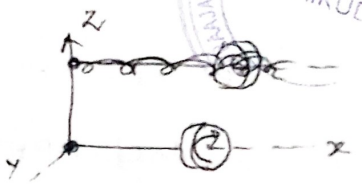
$I = mk^2 = 8 \times (0.15)^2 = 0.18\ kg\,m^2$

$\omega = \frac{2\pi \times 240}{60} = 25.13\ rad/s.$

$C = I\omega\omega_p$

$= 0.18 \times 25.13 \times \omega_p$

$= mg \cdot l$



$\Rightarrow 0.18 \times 25.13 \times \omega_p = 8 \times 9.81 \times 0.2$

$\Rightarrow \boxed{\omega_p = 3.47\ rad/s.}$

Q.4 A disc of radius of gyration 60 mm and mass of 4 kg. is mounted centrally on a horizontal axle of 80 mm length betn the bearings. It spins about the axle at 800 rpm counterclockwise when viewed from the right hand side bearing. The axle precesses about vertical axis at 50 rpm in clockwise direction when viewed from above. Determine the resultant reaction at each bearing due to the mass and gyroscopic effect.

$K = 60\ mm = 0.06\ m$ $\quad m = 4\ kg.$ $\quad N = 800\ rpm$

$l = 80\ mm = 0.08\ m.$ $\quad N_p = 50\ rpm.$

Now $I = mK^2$

$= 4 \times 0.06^2 = 0.0144\ kg\,m^2$

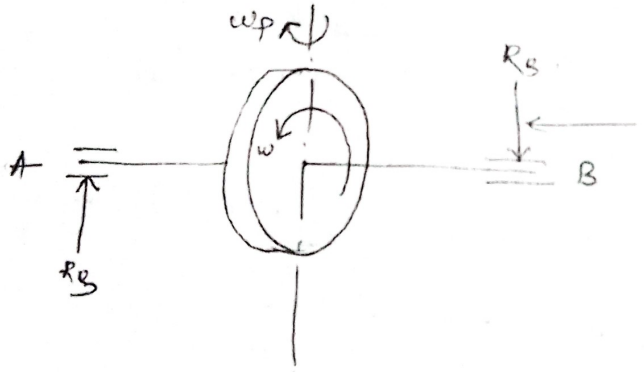
$\omega = \frac{2\pi \times 800}{60} = 83.78\ rad/s.$

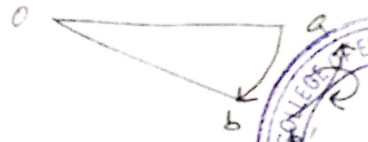
$\omega_p = \frac{2\pi \times 50}{60} = 5.24\ rad/s.$

$C = I\omega\omega_p$

$= 0.0144 \times 83.78 \times 5.24$

$= 6.32\ N\,m.$





The active couple acting the disc is anticlockwise when viewed from the end and the reactive couple is clockwise when viewed from the top.

Force at bearing A due to gyroscopic couple

$= \frac{C}{l} = \frac{6.32}{0.08} = 79\ N$ (upward)

Force at bearing B due to gyroscopic couple

bearing due to wt

= 19.6 N upward)

reaction at A = 79 + 19.6 = 98.6 N (upward)

Reaction at B = 79 − 19.6 = 59.4 N (downward)

3 An aeroplane flying at 240 km/hr turns towards left and completes a quarter circle of radius 60 m. The mass of rotary engine and propeller plane amounts to 450 kg with a radius of gyration of 320 mm. The engine speed is 2000 rpm clockwise when viewed from the rear. Determine the gyroscopic couple on the aircraft and state it's effect.

450 kg $\quad \omega = \frac{2\pi \times 2000}{60} = 209.4$ rad/s

0.32 m $\quad v = \frac{240 \times 1000}{3600} = 66.67$ m/s.

$mk^2 = 450 \times (.32)^2 = 46.08$ kgm^2

$= \frac{v}{r} = \frac{66.67}{60} = 1.11$ rad/s.

$I\omega\omega_p = 46.08 \times 209.4 \times 1.11$

$= 10713$ Nm $= \boxed{10.713 \text{ kNm}}$

~~of an Automobile~~ –

1

2) having a mass of 20 kg, and gyration of 300 mm is given a spin about its axis which is horizontal suspended at the point 250 mm stance of rotation of the flywheel. of precession.

force at each bearing due to the wt of the disc = $\frac{4 \times 9.81}{2} = 19.6\,N$.

Reactive force at each bearing due to wt = 19.6 N (upward)

So reaction at A = 79 + 19.6 = 98.6 N (upward)

Reaction at B = 79 − 19.6 = 59.4 N (downward)

Q-3 An aeroplane flying at 240 km/hr turns towards left and completes a quarter circle of radius 60 m. The mass of rotary engine and propeller plane amounts to 450 kg with a radius of gyration of 320 mm. The engine speed is 2000 rpm clockwise when viewed from the rear. Determine the gyroscopic couple on the aircraft and state it's effect.

$m = 450\,kg$ $\qquad \omega = \frac{2\pi \times 2000}{60} = 209.4\,rad/s$

$K = 0.32\,m$ $\qquad v = \frac{240 \times 1000}{3600} = 66.67\,m/s$.

$I = mK^2 = 450 \times (.32)^2 = 46.08\,kgm^2$

$\omega_p = \frac{v}{r} = \frac{66.67}{60} = 1.11\,rad/s$.

$C = I\omega\omega_p = 46.08 \times 209.4 \times 1.11$

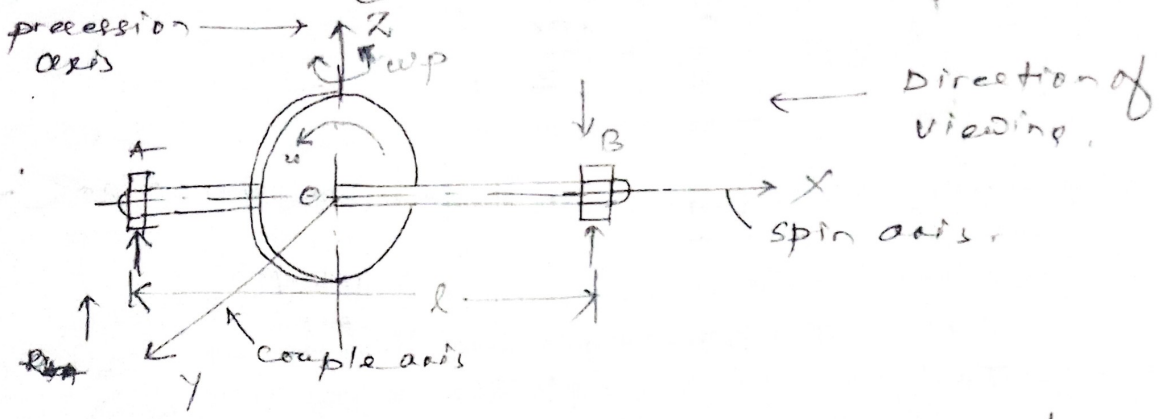
$= 10713\,Nm = \boxed{10.713\,kNm}$.

~~Stability of an Automobile~~ –

Assignment

Q-1 A flywheel having a mass of 20 kg and radius of gyration of 300 mm is given a spin of 500 rpm about its axis which is horizontal. The flywheel is suspended at a point 250 mm from the plane of rotation of the flywheel.

Effect of gyroscopic couple on bearing



Consider a rotating disc of mass m and radius of gyration K is supported in bearings A and B through the shaft. The disc is rotating in anticlockwise when viewed from the right hand side of bearing B. The shaft AB turns to the right by application of couple and thus causing change of angular momentum.

So, gyroscopic couple $\boxed{C = I.\omega.\omega_p}$

forces at bearings A and B

$$\boxed{F_A = F_B = \frac{C}{l}}$$

Reactions at each bearing due to wt. of disc $R_A = R_B = \frac{mg}{2}$

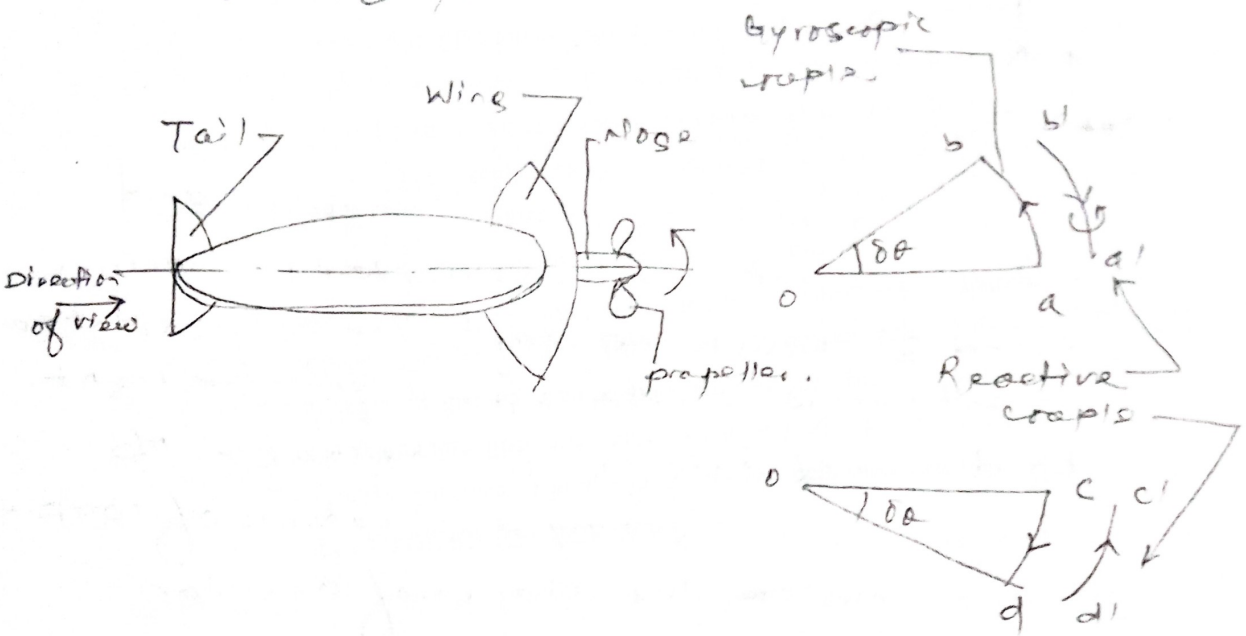
Applying right hand thumb rule in the direction of ab, the direction of thumb will indicate the change of angular momentum, second the direction of fingers will show the direction of couple applied by bearing which is clockwise and the reaction couple will be equal in magnitude and opposite in direction.

$\boxed{D \quad . \quad C \quad . \quad mg}$

Net reaction at B

$$R_{BB} = \frac{c}{l} - \frac{mg}{2}$$

Effect of Gyroscopic couple on Aeroplane:-



An ~~aer~~ aeroplane's engine rotates clockwise when viewed from the rear end. Aeroplane takes turn to the left.

from the angular momentum diagram oa is the angular momentum vector before turning.

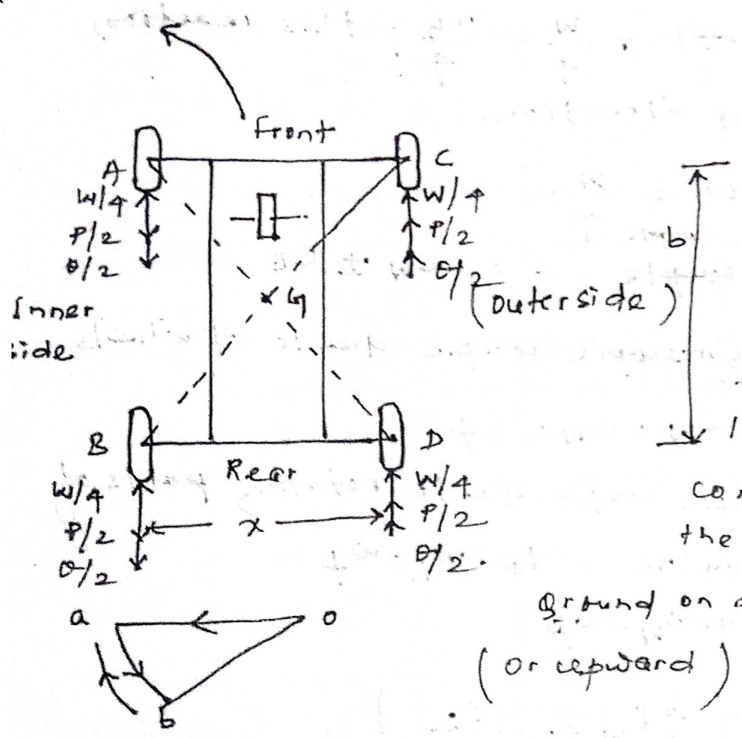
ob = angular momentum vector after turning

ab = active gyroscopic couple

b'a' = reactive "

Now applying right hand thumb rule in looking and ~~thumb being in right~~ the direction of ~~ab~~ ab. The direction of active gyroscopic couple will be clockwise, so reactive gyroscopic couple will be anticlockwise.

Stability of an automobile (while moving in a curved path) :-



Consider a four wheel automobile taking turn towards leftside. A and B are the inner and C and D are the outer wheels while taking the turn it is essential that no wheel is lifted off the ground. The condition is fulfilled as long as the vertical reaction of the ground on any wheel is possitive (or upward).

Let W = wt. of the vehicle in N = $m.g$

m = mass of the vehicle in kg.

x = width of the track in m.

b = wheel base in m.

R = Radius of curvature.

h = distance of centre of gravity above the road surface in m.

r_w = radius of each wheel in m.

I_w, I_E = mass moment of inertia of each wheel and rotating part of engine in $kg\text{-}m^2$ respectively.

ω_p = angular velocity of precession in r/s

$= \frac{v}{R}$

v = linear velocity of vehicle in m/s

ω_W = angular velocity of wheels = $\frac{v}{r}$

ω_E = angular velocity of rotating parts of engine

G = gear ratio = $\frac{\omega_E}{\omega_W}$

Reaction due to weight of the vehicle :-

W of the vehicle is distributed over four wheels equally.

so load on each wheel $= \frac{W}{4} = \frac{mg}{4}$; this reaction will be in upward direction.

thus road reaction $= \frac{W}{4}$

Reaction due to Gyroscopic couple

Total gyroscopic couple $C = C_W \pm C_E$

where C_W = gyroscopic couple due to 4 wheels

$= 4 I_W \cdot \omega_W \cdot \omega_p$

C_E = gyroscopic couple due to rotating parts of the engine $= I_E \cdot \omega_E \cdot \omega_p$

$= I_E \cdot G \cdot \omega_W \cdot \omega_p$

so $\boxed{C = \omega_W \cdot \omega_p \left(4 I_W \pm G I_E \right)}$

+ve sign is taken when the engine and the vehicle wheel rotate in same direction

This gyroscopic couple produces reaction on the road surface. This reaction is +ve on the outer and -ve on the inner wheels. Let the magnitude of this reaction on each two outer and inner wheels be $P/2$.

so $\boxed{\frac{P}{2} = \frac{C}{2x}}$

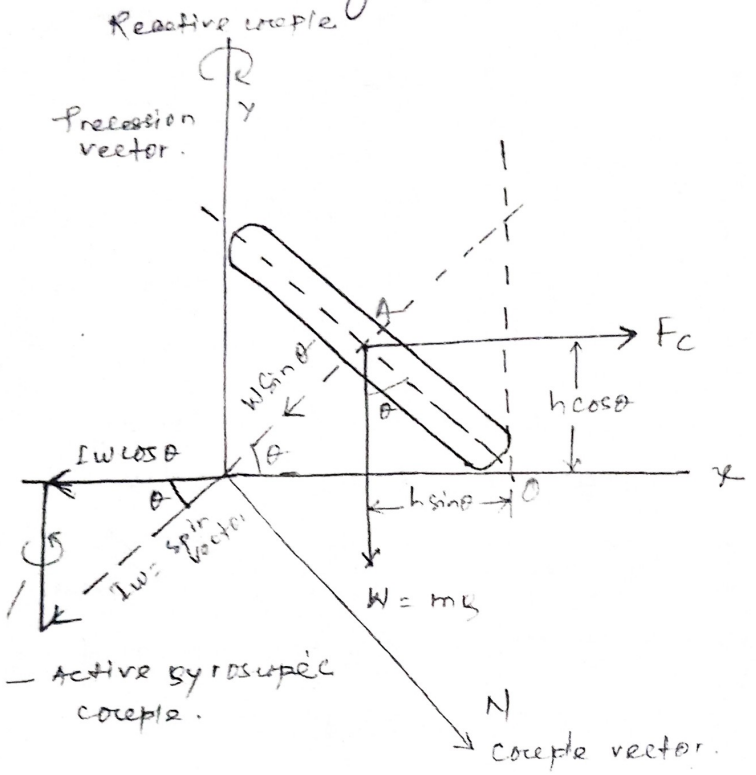
If C value is -ve, the direction of reactions on wheels will be reversed i.e it will be -ve on the outer wheel and +ve on the inner wheel.

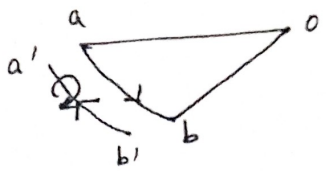
Reaction due to centrifugal effect :-

Vehicle while taking the turn will try to overturn because of the centrifugal effect. The magnitude of centrifugal force is

$\boxed{F = W \cdot \frac{v^2}{}}$

Stability of a two-wheeled vehicle :-





Consider a two wheel vehicle as shown in the figure, taking a left turn.

Let I_W, I_E = mass moment of inertia of each wheel and rotating part of engine respectively in $kg\,m^2$

ω_W, ω_E = angular velocity of wheel and engine in rad/sec respectively.

G = Gear ratio = $\frac{\omega_E}{\omega_W}$

v = linear velocity of the vehicle in m/s

h = height of centre of gravity of the vehicle and rider in N

r_W = radius of each wheel in m.

R = radius of curvature in m.

θ = angle of inclination of wheel to the vertical

$\omega_p = \frac{v}{R}$

$$\boxed{f_H = 2m\omega^2 r \cos\theta \cdot \cos^2\alpha}$$

Similarly total primary force along y axis

$$= m\omega^2 r \cos(\theta-\alpha)\sin\alpha - m\omega^2 r \cos(\theta+\alpha)\sin\alpha$$

$$= m\omega^2 r \sin\alpha\left[\cos(\theta-\alpha) - \cos(\theta+\alpha)\right]$$

$$\boxed{f_V = 2m\omega^2 r \sin^2\alpha \sin\theta}$$

Resultant primary force $f_p = \sqrt{f_H^2 + f_V^2}$

$$\boxed{= 2m\omega^2 r \sqrt{(\cos\theta \cdot \cos^2\alpha)^2 + (\sin\theta \cdot \sin^2\alpha)^2}}$$

Now deriving the expression for secondary force:

secondary force for cylinder 1 along line of stroke OB

$$= \frac{m\omega^2 r}{n} \cos 2(\theta-\alpha)$$

secondary force for cylinder 1 along x-axis

$$f_{H_3} = \frac{m\omega^2 r}{n} \cos 2(\theta-\alpha)\cos\alpha$$

secondary force of cylinder 2 along the line of stroke OC

$$= \frac{m\omega^2 r}{n} \cos 2(\theta+\alpha)$$

secondary force of cylinder 2 along x-axis

$$f_{H_4} = \frac{m\omega^2 r}{n} \cos 2(\theta+\alpha)\cos\alpha$$

Thus total secondary force along x-axis

$$f_{H_s} = f_{H_3} + f_{H_4}$$

$$= \frac{m\omega^2 r}{n} \cos 2(\theta-\alpha)\cos\alpha + \frac{m\omega^2 r}{n} \cos 2(\theta+\alpha)\cos\alpha$$

$$= \frac{m\omega^2 r}{n} \cos\alpha\left[\cos 2(\theta-\alpha) + \cos 2(\theta+\alpha)\right]$$

$$= \frac{m\omega^2 r}{n} \cos\alpha \cdot 2\cos 2\alpha \cdot \cos 2\theta$$

Similarly total primary force along y axis

$= m\omega^2 r \cos(\theta-\alpha)\sin\alpha - m\omega^2 r \cos(\theta+\alpha)\sin\alpha$

$= m\omega^2 r \sin\alpha\left[\cos(\theta-\alpha) - \cos(\theta+\alpha)\right]$

$$\boxed{F_V = 2m\omega^2 r \sin^2\alpha \sin\theta}$$

Resultant primary force $F_P = \sqrt{F_H^2 + F_V^2}$

$$\boxed{= 2m\omega^2 r \sqrt{(\cos\theta \cdot \cos^2\alpha)^2 + (\sin\theta \cdot \sin^2\alpha)^2}}$$

Now deriving the expression for secondary force:

secondary force for cylinder along line of stroke OB

$= \frac{m\omega^2 r}{n} \cos 2(\theta-\alpha)$

secondary force for cylinder 1 along x-axis

$F_{H_3} = \frac{m\omega^2 r}{n} \cos 2(\theta-\alpha)\cos\alpha$

secondary force of cylinder 2 along the line of stroke OC

$= \frac{m\omega^2 r}{n} \cos 2(\theta+\alpha)$

secondary force of cylinder 2 along x-axis

$F_{H_4} = \frac{m\omega^2 r}{n} \cos 2(\theta+\alpha)\cos\alpha$

Thus total secondary force along x-axis

$F_{H_S} = F_{H_3} + F_{H_4}$

$= \frac{m\omega^2 r}{n} \cos 2(\theta-\alpha)\cos\alpha$

$+ \frac{m\omega^2 r}{n} \cos 2(\theta+\alpha)\cos\alpha$

$= \frac{m\omega^2 r}{n} \cos\alpha\left[\cos 2(\theta-\alpha) + \cos 2(\theta+\alpha)\right]$

$= \frac{m\omega^2 r}{n} \cos\alpha \cdot 2\cos 2\alpha \cdot \cos 2\theta$

$$= \boxed{\frac{2m\omega^2 r}{n} \cos 2\alpha \cdot \cos 2\theta \cdot \cos\alpha}$$

Similarly total secondary force along y-axis

$$F_{VS} = \frac{2m\omega^2 r}{n} \sin 2\alpha \cdot \sin 2\theta \cdot \sin\alpha$$

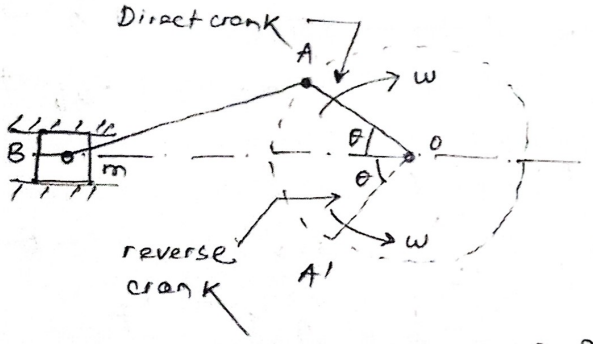
Resultant force F_S

$$F_S = \sqrt{F_{HS}^2 + F_{VS}^2}$$

$$= \boxed{\frac{2m\omega^2 r}{n} \sqrt{(\cos 2\alpha \cdot \cos 2\theta \cdot \cos\alpha)^2 + (\sin 2\alpha \cdot \sin 2\theta \cdot \sin\alpha)^2}}$$

Direct and Reverse crank Method of Balancing

This method of balancing is useful for balancing of radial or V-engines. connecting rods are attached to a common crank. In this case plane of rotation of crank is same, so there is no unbalanced primary or secondary couple. Only the primary and secondary forces are to be balanced.



As shown in the figure in the reciprocating engine the crank OA rotates uniformly at ω rad/s in clockwise direction.

Let OA makes an angle θ with OB at any instant. The reverse crank OA' is the image of direct crank OA. and it rotates in anti clockwise direction by the gear mechanism. OA and OA' are called as direct and reverse cranks respectively. The direct and reverse cranks coincide at I.D.C (inner dead centre). Let's say mass of reciprocating parts at B is 'm'.

The primary force can be obtained which is balanced

[illegible]

$$F_{x_2} = \frac{2mr\omega^2}{n} \cos 2\theta \quad \cos 4\theta \quad \cos 6\theta \quad \cos 8\theta$$

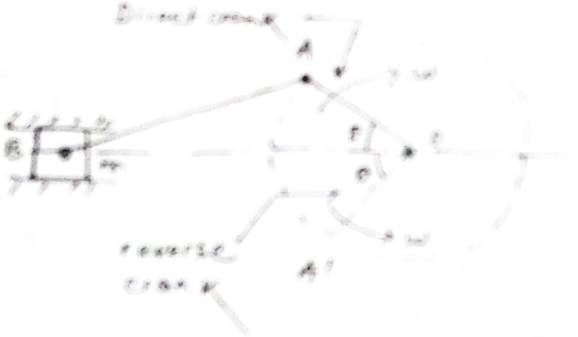
Resultant force F_2

$$F_2 = \sqrt{F_{y_2}^2 + F_{x_2}^2}$$

$$= \boxed{\frac{2mr\omega^2}{n} \sqrt{(\cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta)^2 + (\sin 2\theta + \sin 4\theta + \sin 6\theta + \sin 8\theta)^2}}$$

Direct and Reverse crank method of Balancing

This method of balancing is useful for balancing of radial or V-engines connecting rods are attached to a common crank. In this case, plane of rotation of crank is same, so there is no unbalanced primary or secondary couple. Only the primary and secondary forces are to be balanced.

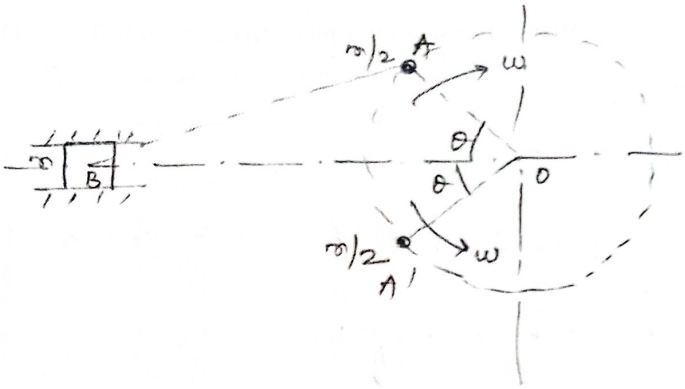


As shown in the figure, in the reciprocating engine the crank OA strokes uniformly about O and is in clockwise direction.

Let OA makes an angle θ with OC at any instant. The reverse crank OA' is the imaginary crank to OA and it strokes in anti clockwise direction by the gear mechanism. OA and OA' are called as direct crank and reverse cranks respectively. The direct and reverse cranks coincide at I.D.C (inner dead centre). Let's say mass of reciprocating parts at B is m.

The primary force can be established which is equal to $m\omega^2 r \cos\theta$. If a mass m is placed at a radius r in it produces centrifugal force of

magnitude $m\omega^2 r$. The horizontal component of this force is $m\omega^2 r \cos\theta$ which is equal to the primary force. Thus both the forces are equal and balanced horizontally only.



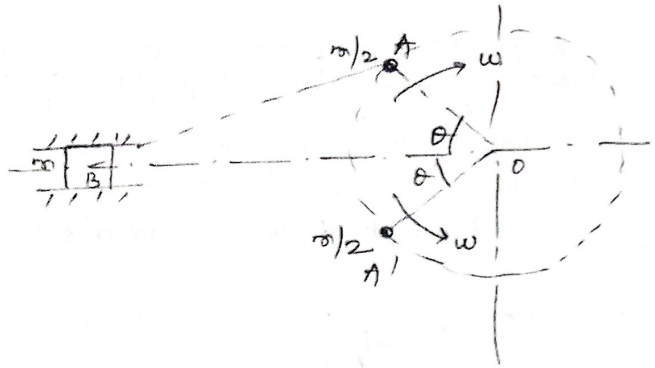
Now let's assume that mass 'm' is divided equally and placed at A and A' as shown in the figure.

The horizontal component of centrifugal forces due to masses $m/2$ placed at A and A' will be $m/2\, \omega^2 r \cos\theta$ each. Their combined effect will be equal to primary force

$$\underset{(m/2 \text{ at } A)}{m/2\, \omega^2 r \cos\theta} + \underset{(m/2 \text{ at } A')}{m/2\, \omega^2 r \cos\theta} = F_p$$

In this case, we have put each revolving mass equal to one half of the reciprocating mass to determine the primary force. The components of centrifugal forces of two masses normal to the line of stroke will be equal and opposite i.e. one upwards and other downwards. So vertical component

this force is $m\omega^2 r \cos\theta$ which is equal to the primary force. Thus both the forces are equal and balanced horizontally only.



Now let's assume that mass 'm' is divided equally and placed at A and A' as shown in the figure.

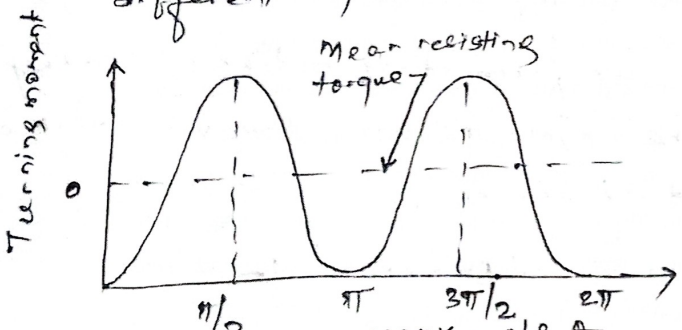
The horizontal component of centrifugal force due to masses $m/2$ placed at A and A' will be $\frac{m}{2}\omega^2 r\cos\theta$ each. Their combined effect will be equal to primary force

$$\underset{(m/2 \text{ at } A)}{\frac{m}{2}\omega^2 r\cos\theta} + \underset{(m/2 \text{ at } A')}{\frac{m}{2}\omega^2 r\cos\theta} = \underset{F_p}{m\omega^2 r\cos\theta}$$

In this case, we have put each revolving mass equal to one half of the reciprocating mass to determine the primary force. The components of centrifugal forces of two masses normal to the line of stroke will be equal and opposite i.e. one upwards and other downwards. So vertical component also balance each other.

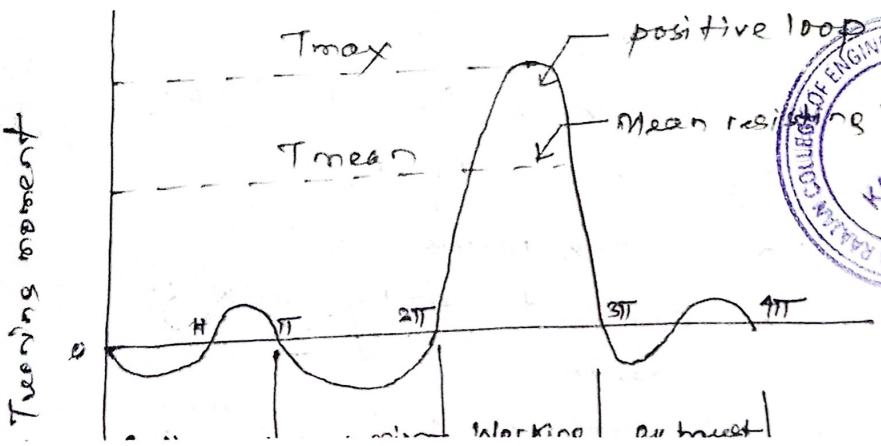
From the above expression it is clear that the turning moment T varies with the variation of crank rotation angle θ. If the value of T is plotted against crank angle θ in radians, the diagram obtained is called turning moment diagram. The turning moment diagram (or crank effort diagram) is the graphical representation of 'T' for various positions of crank (θ).

- The area of turning moment diagram, for a particular cycle represents the work done during that cycle of operation. There are different turning moment diagram for different cycles as shown in the figure.



(Single cylinder double acting steam engine)

(A)



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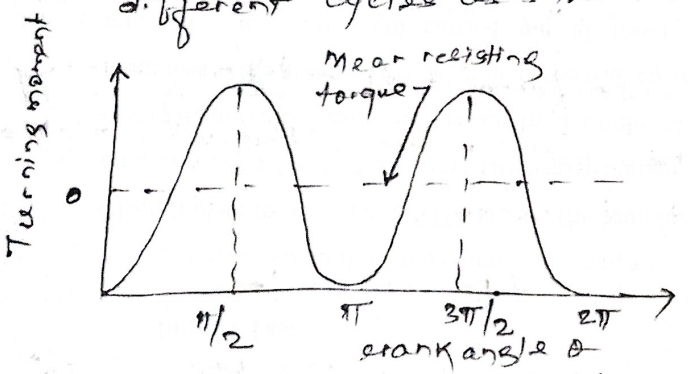
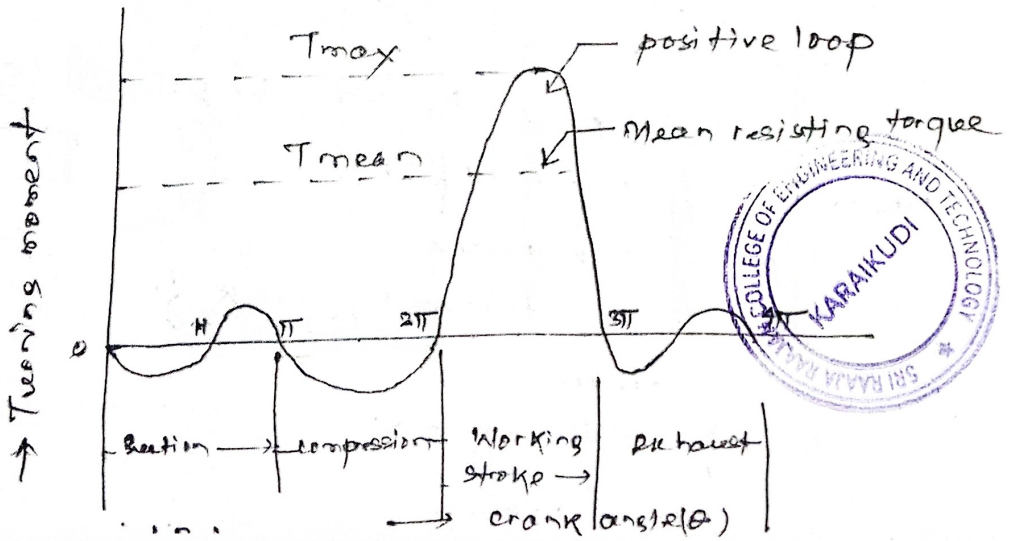
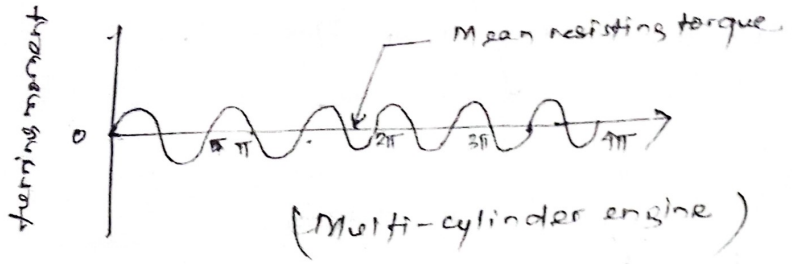


(Single cylinder double acting steam engine)

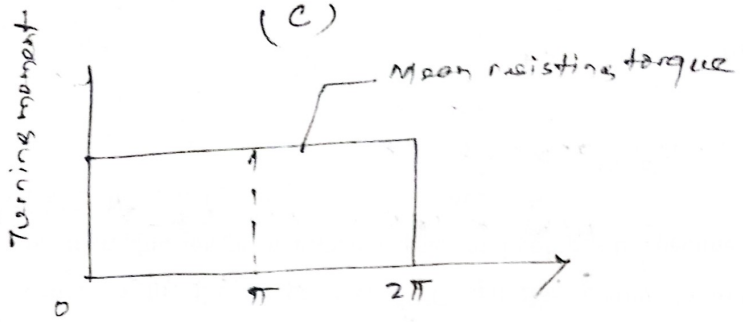
(A)





(Multi-cylinder engine)

(C)

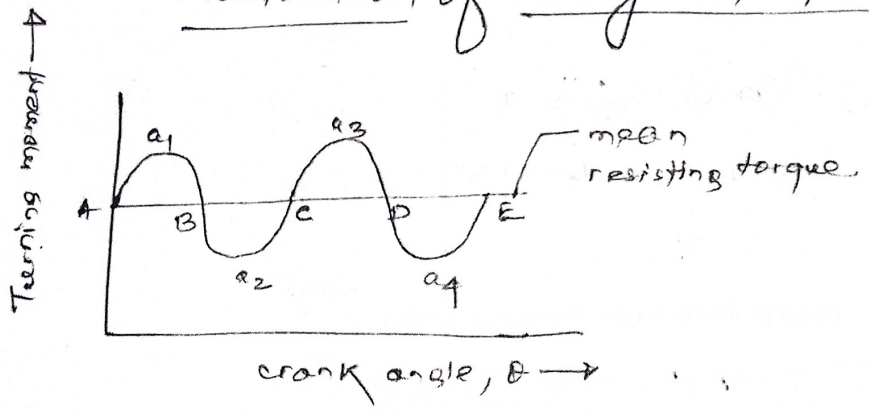


~~crank~~ (D)

For each cycle of operation the workdone by the engine must be at least equal to the work required to overcome the load which is called mean resisting torque. If the work done by the engine is more than the mean resisting torque, the engine will accelerate. On the other hand the engine will retard or stop if the workdone is less than the resisting torque.

- As shown in fig (A) during suction stroke, pressure inside the cylinder is less than atmospheric pressure so in majority a negative loop is formed. During compression stroke, the work is done on the gas so a higher negative loop is formed. In the working stroke work is done by gas, so a large positive energy loop is obtained. During exhaust stroke the loop is negative as the work is done on the gas.

Fluctuation of Energy and Speed :-



The above figure represents the turning moment diagram of a multi-cylinder engine. Let AE is the mean resisting torque line. Let the area of energy loop between crank rotation from A to B, B to C, C to D and D to E, are a_1, a_2, a_3 and a_4 respectively.

Area of energy loop below the mean resisting torque line is taken with -ve sign and that of above the mean resisting torque line with +ve sign. The variation of energy above and below the mean resisting torque line is called ~~mean~~ fluctuation of energy. This fluctuation of energy may be obtained by turning moment diagram for one complete cycle of operation.

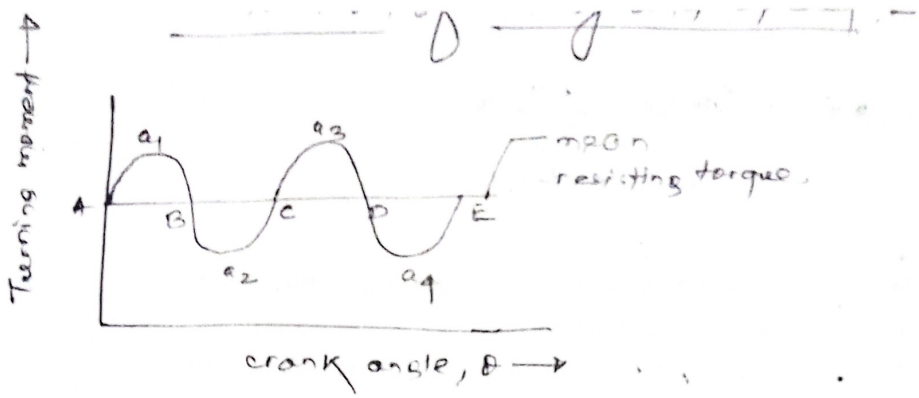
Let the energy at point A = U

at point B = $U + a_1$

at point C = $U + a_1 - a_2$

at point D = $U + a_1 - a_2 + a_3$

at point E = $U + a_1 - a_2 + a_3 - a_4$



The above figure represents the turning moment diagram of a multi-cylinder engine. Let AE is the mean resisting torque line. Let the area of energy loop between crank rotation from A to B, B to C, C to D and D to E; are a_1, a_2, a_3 and a_4 respectively.

Area of energy loop below the mean resisting torque line is taken with -ve sign and that of above the mean resisting torque line with +ve sign. The variation of energy above and below the mean resisting torque line is called ~~mean~~ fluctuation of energy. This fluctuation of energy may be obtained by turning moment diagram for one complete cycle of operation.

Let the energy at point A = U

at point B = $U + a_1$

at point C = $U + a_1 - a_2$

at point D = $U + a_1 - a_2 + a_3$

at point E = $U + a_1 - a_2 + a_3 - a_4$

Difference betn max^m and min^m energy is called max^m fluctuation of energy (E_f)

So $\boxed{E_f = \text{max}^m \text{ energy} - \text{min}^m \text{ energy}}$

Coefficient of fluctuation of energy :-

Coefficient of fluctuation of energy

$$\boxed{U_f = \frac{\text{Max}^m \text{ fluctuation of energy}}{\text{workdone / cycle}}}$$

Coefficient of Fluctuation of Speed :-

Coefficient of fluctuation of speed is indicated by C_f

$$C_f = \frac{N_1 - N_2}{N} = \frac{N_1 - N_2}{\left(\frac{N_1 + N_2}{2}\right)} = \frac{2\pi(\omega_1 - \omega_2) \times 2}{2\pi(\omega_1 + \omega_2)}$$

$$\Rightarrow \boxed{C_f = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}}$$

The reciprocal of coefficient of fluctuation of speed is known as coefficient of steadiness and is denoted by m

$$\boxed{m = \frac{1}{C_f}}$$

$N_1 \rightarrow$ max^m speed of flywheel

$N_2 \rightarrow$ min^m speed of flywheel

Energy stored in a flywheel :-

Let m = mass of flywheel in kg

K = radius of gyration, m.

moment of inertia of flywheel $I = mK^2$

N_1 and N_2 = max and min speeds of flywheel

Fluctuation of speed = $N_1 - N_2$

mean speed $N = \frac{N_1 + N_2}{2}$

coefficient of fluctuation speed

$$C_f = \frac{N_1 - N_2}{N}$$

Mean energy stored in the flywheel

$$E = \frac{1}{2} I \omega^2$$

Max energy stored by flywheel

$$E_1 = \frac{1}{2} I \omega_1^2 \quad \left(\text{where } \omega_1 = \frac{2\pi N_1}{60} \right)$$

Min energy stored by flywheel

$$E_2 = \frac{1}{2} I \omega_2^2 \quad \left(\text{where } \omega_2 = \frac{2\pi N_2}{60} \right)$$

Max fluctuation of energy, E_f

$$E_f = E_1 - E_2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2) \cdot \frac{(\omega_1 - \omega_2)}{\left(\frac{\omega_1 + \omega_2}{2}\right)} \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= \frac{1}{2} I \left(\frac{\omega_1 + \omega_2}{2}\right) \cdot C_f \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= I \cdot C_f \left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$$= I \cdot C_f \cdot \omega^2$$

$$= \frac{1}{2} I \omega^2 \cdot (C_f \cdot 2)$$

$$= \left(\frac{1}{2} I \omega^2\right) 2 C_f$$

moment

N_1 and N_2 = max and min speeds of flywheel

Fluctuation of speed = $N_1 - N_2$

mean speed $N = \frac{N_1 + N_2}{2}$

coefficient of fluctuation speed

$$C_f = \frac{N_1 - N_2}{N}$$

Mean energy stored in the flywheel

$$E = \frac{1}{2} I \omega^2$$

Max energy stored by flywheel

$$E_1 = \frac{1}{2} I \omega_1^2 \quad \left(\text{where } \omega_1 = \frac{2\pi N_1}{60} \right)$$

Min energy stored by flywheel

$$E_2 = \frac{1}{2} I \omega_2^2 \quad \left(\text{where } \omega_2 = \frac{2\pi N_2}{60} \right)$$

Max fluctuation of energy, E_f

$$E_f = E_1 - E_2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2) \cdot \frac{(\omega_1 - \omega_2)}{\left(\frac{\omega_1 + \omega_2}{2}\right)} \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= \frac{1}{2} I \left(\frac{\omega_1 + \omega_2}{2}\right) \cdot C_f \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= I \cdot C_f \left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$$= I \cdot C_f \cdot \omega^2$$

$$= \frac{1}{2} I \omega^2 \cdot (C_f \cdot 2)$$

$$= \left(\frac{1}{2} I \omega^2\right) 2 C_f$$

$$\Rightarrow \boxed{E_f = 2 I C_f}$$

Now $\boxed{\text{workdone/cycle} = \frac{P \times 60}{n}}$

where P = Power transmitted in watt.

n = no of working strokes/minute.

= $N/2$ in case of 4-s IC engine

= N in case of steam engine and 2-s IC engines.

moment of inertia of solid flywheel disc

$$\boxed{I = m \cdot \frac{D^2}{8}}$$

mass of flywheel rim

$$\boxed{m = A \cdot \pi D \cdot \rho}$$

and $\sigma = \rho \cdot v^2$

$$\Rightarrow \boxed{\sigma = \rho \cdot \left(\frac{\pi D N}{60}\right)^2}$$

area of flywheel $\boxed{A = b \times t}$

Where A = cross-sectional area of flywheel rim

b = width of rim

t = thickness of rim.

ρ = density of rim material kg/m^3

D = mean diameter of rim in m

v = linear velocity of flywheel in m/s

N = speed of flywheel in rpm.

The load is max^m when punching operation takes place and it is zero during the rest of the cycle. Thus there are high variations in load and hence there is high fluctuations in speed.

The speed of crank increases during idle operations and decreases during actual operation. The flywheel absorbs energy in the form of kinetic energy during idle period and makes up for the energy requirements during actual operation of punching. For this purpose a flywheel of large size is selected which keeps the speed fluctuation within permissible limits.

Analysis of Flywheel in punching operation

Let E_1 = energy required in punching operation

σ_s = max^m shear stress of plate material

d = dia. of hole to be punched

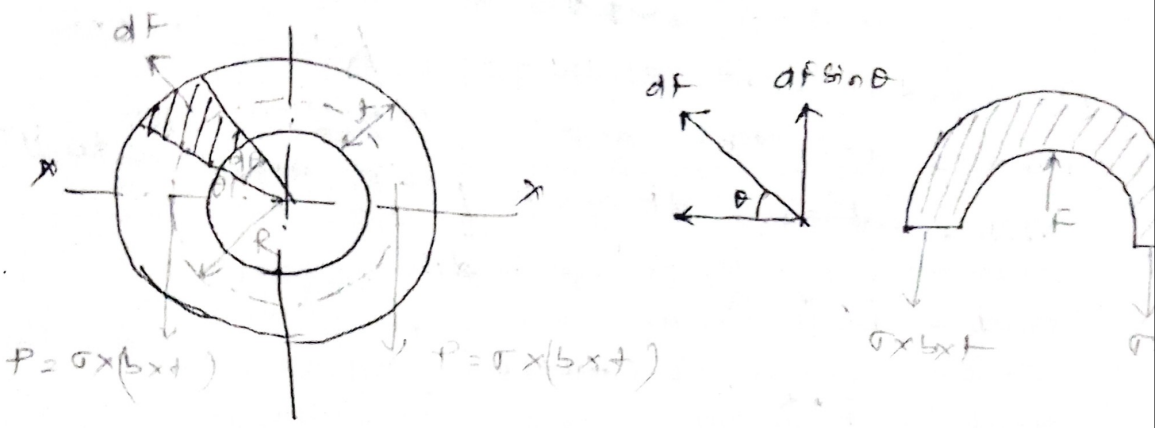
t = thickness of plate.

Max^m shear force

$$F_s = \sigma_s \times \pi d t \quad \text{--- (1)}$$

Assuming that during punching operation shear force decreases from max^m value to zero value.

Dimensions of flywheel Rim



Let D = mean diameter of rim, m

R = mean radius of rim, m

A = cross-sectional area of rim, m^2

N = speed of flywheel, rpm

$\omega = \frac{2\pi N}{60}$ rad/s

v = linear velocity at the mean radius, m/s

$= \omega R = \frac{\pi D N}{60}$

m = mass of element, kg;

σ = tensile or hoop stress due to centrifugal force, N/m^2

ρ = density of rim material, kg/m^3

consider a small element of the rim as shown in the shaded portion in the fig. Let the small element substend an angle dθ at the centre of the flywheel. The centrifugal force dF acts radially outwards. The force dF acting on the centre of the element is given by

$$dF = \frac{mv^2}{R} = m\omega^2 R$$

$$= \rho(R\,\delta\theta\,A)\,\omega^2 R = \rho A R^2 \omega^2 \delta\theta$$

vertical component of dF

$= dF \sin\theta = \rho A R^2 \omega^2 \sin\theta \, d\theta$

Now total vertical upward force on the rim across the x-x is obtained by integrating $dF \sin\theta$ from angle 0 to π

$$F = \rho A R^2 \omega^2 \int_0^{\pi} \sin\theta \, d\theta$$

$$= \rho A R^2 \omega^2 (-\cos\theta)_0^{\pi}$$

$$\Rightarrow \boxed{F = 2\rho A R^2 \omega^2} \quad \text{--- (1)}$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress) and it is resisted by $2P$, such that

$$2P = 2\sigma A \quad \text{--- (2)}$$

Equating (1) and (2)

$$2\sigma A = 2\rho A R^2 \omega^2$$

$$\Rightarrow \sigma = \rho R^2 \omega^2 = \rho v^2$$

$$\Rightarrow \boxed{v = \sqrt{\frac{\sigma}{\rho}}}$$

We know that mass of the rim

$$\boxed{m = \text{volume} \times \text{density} = \pi D A \rho}$$

or area $\boxed{A = \frac{m}{\pi D \rho}}$

For rectangular cross-sectional area of rim $\boxed{A = b \times t}$

The average energy of punching one hole

$$E_1 = \frac{1}{2} F_s \cdot t$$

Let E_2 = energy supplied by motor during punching a hole / revolution

$$= E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right)$$

Max^m fluctuation of energy

$$E_f = E_1 - E_2$$
$$= E_1 - E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right)$$

If s is the stroke length of punch, then punch moves 2s in one revolution

$$\text{so} \quad \frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}$$

where r = crank radius.

Above relation is obtained by calibration of crank motion with punch motion.

Q.1 A flywheel of a steam engine weighs 2000N and has a radius of gyration of 76cm. The starting torque of steam engine is 130 kg. and m is assumed to be constant. Determine the angular acceleration of flywheel along with speed and kinetic energy after 10s.

Given data:- K = 0.76 m

starting torque T = 130 kgm
= 130 × 9.81 = 1274 N.m

mass of flywheel $m = \frac{W}{g} = \frac{2000}{9.81} = 204.08$ kg

moment of inertia of flywheel

$$I = mK^2 = 204.08 \times (0.76)^2 = 117.88 \text{ kgm}^2$$

angular acceleration of flywheel (α)

We have $T = I \cdot \alpha$

$$\Rightarrow \alpha = \frac{T}{I} = \frac{1274}{117.88} = 10.8 \text{ rad/s}^2$$

The flywheel started from rest, so $\omega_1 = 0$

and $\omega_2 = \omega_1 + \alpha t$

$$= 0 + 10.8 \times 10 = 108 \text{ rad/s.}$$

Kinetic energy $K.E = \frac{1}{2} I \omega_2^2$

$$= \cancel{\frac{1}{2} \times 1274 \times} \frac{1}{2} \times 117.88 \times 108^2.$$

$$= \boxed{687476.16 \text{ N-m.}}$$ (Ans)

Introduction:-

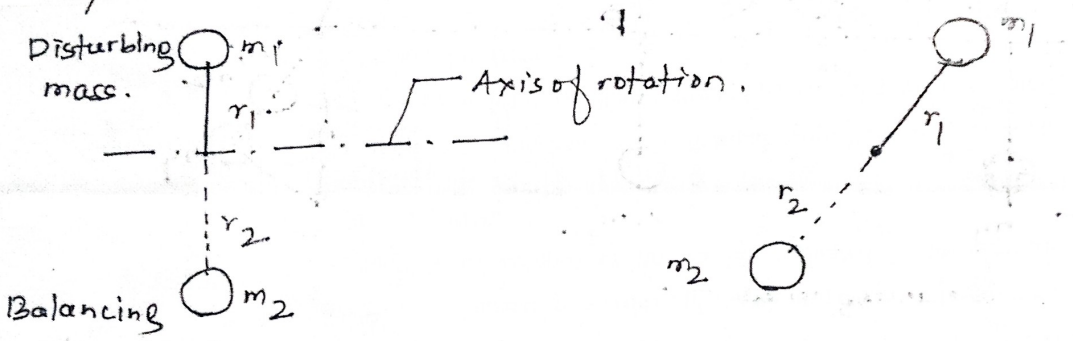
Machines have several rotating parts. Some of them have reciprocating motion e.g. piston and some of them have rotating motion e.g. crankshaft. If these moving parts are not in complete balance, inertia force generation would lead to vibration, noise, wear and tear of the parts.

- Balancing plays a major role in designing these systems to reduce unbalance to an acceptable limit.

Balancing of Single Revolving mass:-

(i) Balancing in same plane (ii) balancing in different plane.

(i) Balancing and disturbing mass revolve in same plane:-



Let m_1 = mass attached to the shaft
ω = angular velocity of the mass in rad/s.
r_1 = distance of C.G. of the mass from axis of rotation.

In order to counteract the disturbing force e.g. the centrifugal force due to m_1, a counter mass m_2 at a radius r_2 is placed in the same plane, such that the centrifugal forces due to the two masses are equal and opposite.

Mathematically $F_{c_1} = m_1\omega^2 r_1$

balancing force $F_{c_2} = m_2\omega^2 r_2$

For balancing $F_{c_1} = F_{c_2}$

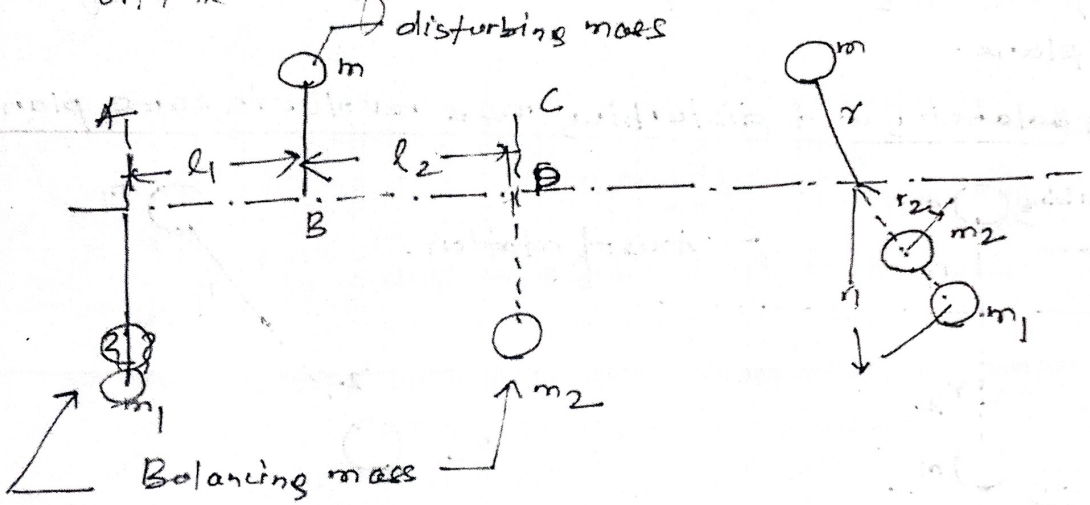
$\Rightarrow m_1\omega^2 r_1 = m_2\omega^2 r_2$

Generally the values of [illegible]

value of balancing mass m_2

(ii) Balancing and Disturbing mass revolve in different plane:—

In case the balancing and the disturbing mass lie in different planes, the disturbing mass can not be balanced by a single mass as there will be a couple left unbalanced. In such case at least two balancing masses are required for complete balancing. The three masses are arranged in such a way that the resultant force and couple on the shaft are zero.



Let m = mass of disturbing body ~~acting~~ acting in plane B

m_1 = mass of balancing weight acting in plane A

m_2 = mass of balancing weight acting in plane B

l_1 = distance betn plane A and B

l_2 = distance betn plane B and C

$l = l_1 + l_2$

r, r_1, r_2 → distances of CG of m, m_1, m_2 respectively

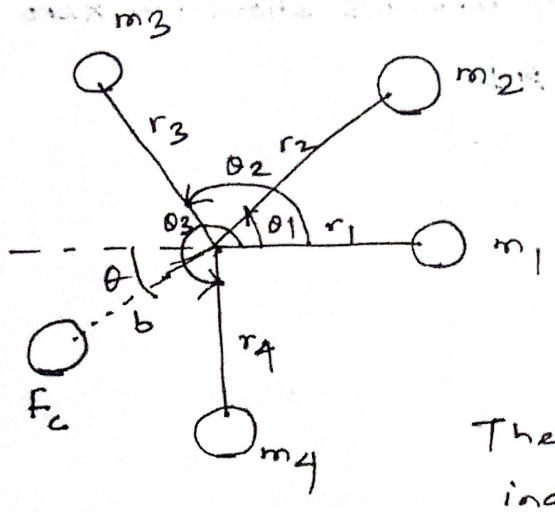
Now $F_c = m\omega^2 r$

$F_{c_1} = m_1\omega^2 r_1$

$F_{c_2} = m_2\omega^2 r_2$

For balancing the centrifugal force of ~~balanc~~ disturbing mass must be equal to the sum of centrifugal force of balancing mass

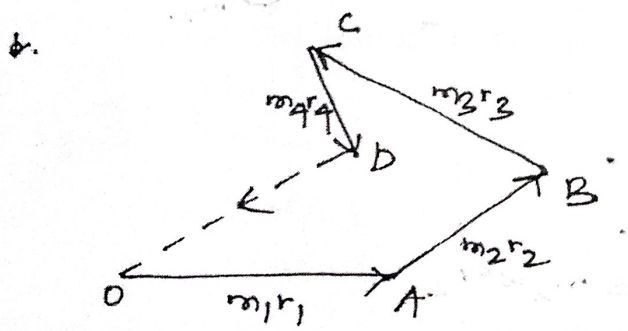
Balancing of several masses revolving in same plane :-



Consider any no of masses say four m_1, m_2, m_3 and m_4 rigidly attached to the shaft and lie in same plane. Let r_1, r_2, r_3 and r_4 be the radii of rotation of masses. Their relative positions are indicated by angles $\theta_1, \theta_2, \theta_3$ and θ_4.

During rotation of shaft, centrifugal force acts radially outward, and will be proportional to m.r. Resultant force may be obtained by drawing the polygon of forces

Polygon method



1. Draw OA parallel to m_1r_1 and magnitude $OA = m_1r_1$
2. From A draw AB parallel and equal to m_2r_2.
3. From B draw BC parallel and equal to m_3r_3
4. from C draw CD parallel and equal to m_4r_4
5. Join D with O, DO represents the direction and magnitude of balanced force.

Analytical method

Resolve each force horizontally and vertical

$\therefore F_H = m_1 r_1 \cos\theta + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$ —— (2)

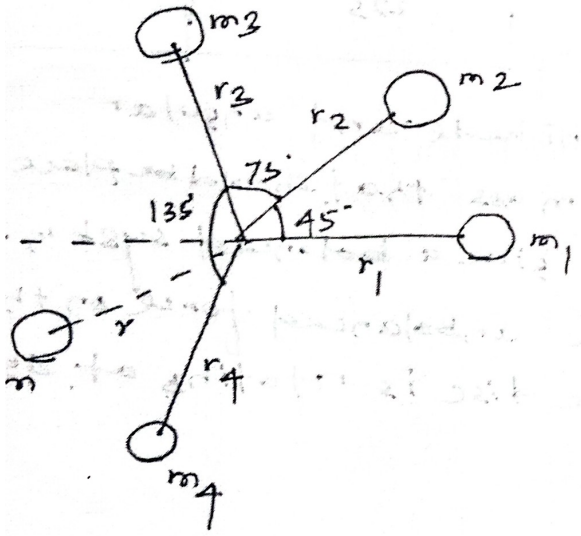
The resultant B.b may be written as

$$\boxed{B.b = \sqrt{(F_V^2 + F_H^2)}}$$

And its direction $\tan\theta = \dfrac{F_V}{F_H}$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)}$$

Q.1 Four masses m_1, m_2, m_3 and m_4 having their radii of rotations as 200 mm, 150 mm, 250 mm and 300 mm are 200 kg, 300 kg, 240 kg and 260 kg. The angle betn the successive masses are 45°, 75° and 125°. Find position and magnitude of the balance mass required if its radius of rotation is 200 mm.



We have

$m_1 = 200$ kg $m_2 = 300$ kg

$m_3 = 240$ kg $m_4 = 260$ kg.

$\theta_1 = 0$ $\theta_2 = 45°$

$\theta_3 = 45 + 75 = 120°$

$\theta_4 = 120° + 135 = 255°$

$r_1 = .2$ m $r_2 = 0.15$ m $r_3 = .25$ m

$r_4 = 0.3$ m.

$r = 0.2$ m

Analytical method

$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120 + 78 \sin 255°$

$= 8.439$ kg-m.

$\Sigma F_H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_2 \cos\theta_4$

$= 40 \cos 0 + 45 \cos 45 + 60 \cos 120 + 78 \cos 255 \ldots$

Resultant force

$$F = \sqrt{F_V^2 + F_H^2} = 23.2 \text{ Kg-m}$$

Now $m.r = 23.2$ Kg m

$$\Rightarrow m = \frac{23.2}{0.2} = \boxed{116 \text{ Kg}}$$

Direction

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

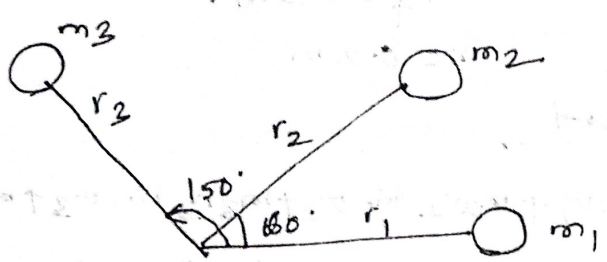
$$\Rightarrow \theta = \tan^{-1}\left(\frac{8.43}{21.63}\right) = \boxed{\cancel{201.5^\circ}\ 21.29^\circ}$$

Direction from m_1 = $180 + 21.29 = 201.5^\circ$ arround a vertical

Q.2 A circular disc rotating around a vertical spindle, has the following masses placed on it.

mass	θ, wrt X-X	Distance from centre (mm)	Magnitude
m_1	0	260	2.5
m_2	60	300	3.5
m_3	150	225	5.0

Determine the magnitude and angular position of a mass that should be placed at 262.5 mm to give a balanced system. Also determine the unbalanced force on the spindle when the disc is rotating at 250 rpm.



$$\Sigma F_V = m_1 r_1 \sin\theta + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$$

$$= 0.65 \sin 0 + 1.05 \sin 60^\circ + 1.125 \sin 150^\circ$$

Resultant force $F = 1.484$ Kgm.

so $m.r = 1.484$ Kgm

$\Rightarrow \boxed{m = 5.653 \text{ Kg.}}$

$\theta = \tan^{-1}\left(\frac{1.471}{0.2007}\right) = \boxed{82.23^\circ}$

Direction from $m_1 = \boxed{262.23^\circ}$

Magnitude of Resultant force.

$m\omega^2 r = 5.653 \times \left(\frac{2\pi \times 250}{60}\right)^2 \times .2625$

$= \boxed{1017 \text{ N.}}$

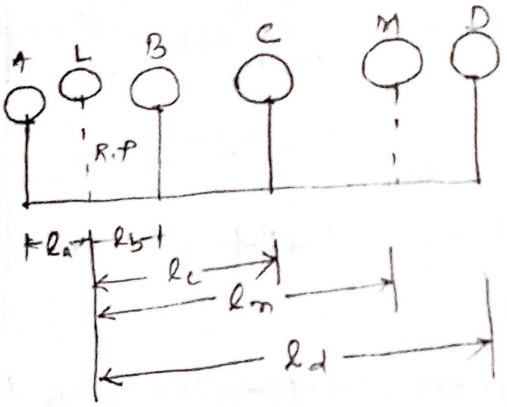
Balancing of Different Planes :-

- Balancing of several masses revolving in different planes is done by transfer of the centrifugal force acting in different planes to a single plane, known as reference plane, thereby masses rotating in different planes are transferred to reference plane.

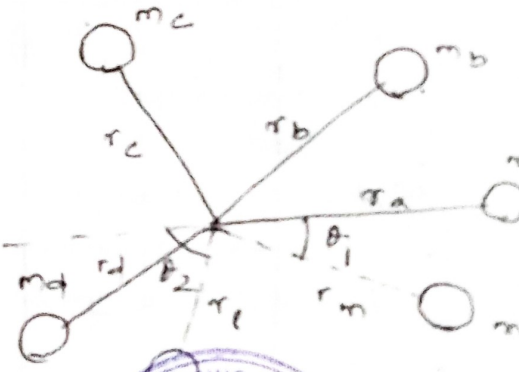
The effect of transferring the rotating mass m in the reference plane is to generate a centrifugal force $F_c = m\omega^2 r$ and a couple $C = F_c \cdot l$ in the reference plane where l = distance betn the reference plane and rotating.

- For complete balancing of such system, two conditions must be satisfied.

1. Resultant centrifugal force must be zero
2. Resultant couple must be zero.



(position of planes)



(Angular position masses)

Let's consider several masses m_a, m_b, m_c and m_d revolving in planes A, B, C and D respectively

Two masses for balancing are used because of the following use:

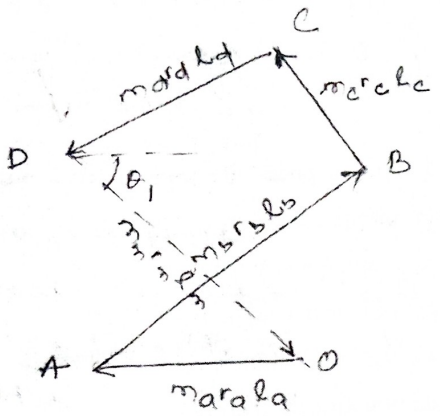
1. If a single mass is used the system will be difficult to handle.
2. If more than two masses are used, no of unknown parameters will be more than no of equations.

Procedure:-

1. Take one plane L as the reference plane. Distance to the left of this plane are taken with minus sign and those to right with +ve sign.
2. Tabulate the forces and couples as shown in the table.

Plane (1)	Mass (m) (2)	radius (r) (3)	centrifugal force ω^2 (mr) (4)	Distance from RP. (l) (5)	couple ω^2 (mrl) (6)
A	m_a	r_a	$m_a r_a$	$-l_a$	$-m_a r_a l_a$
L (RP)	m_l	r_l	$m_l r_l$	0	0
B	m_b	r_b	$m_b r_b$	l_b	$m_b r_b l_b$
C	m_c	r_c	$m_c r_c$	l_c	$m_c r_c l_c$
M	m_m	r_m	$m_m r_m$	l_m	$m_m r_m l_m$
D	m_d	r_d	$m_d r_d$	l_d	$m_d r_d l_d$

3. Draw the couple polygon. Couple $m_a r_a l_a$ is -ve wrt RP. so the couple $(-m_a r_a l_a)$ is drawn radially inwards as it's in reverse direction of Om_a. Couple $m_b r_b l_b$ is +ve wrt RP so it's drawn in the direction of Om_b. Similarly couples $m_c r_c l_c$ and $m_d r_d l_d$ are drawn in the



(couple polygon)

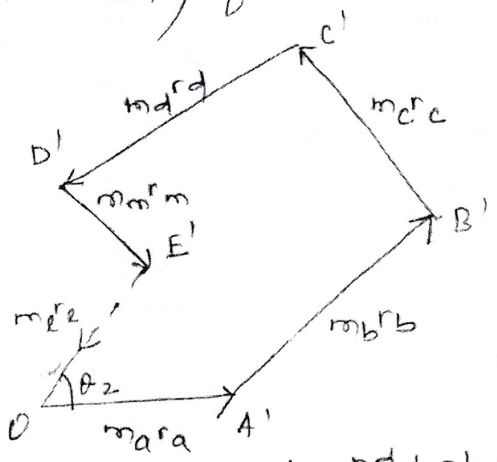
The balancing couple $m_m r_m l_m$ is the closing side. couple OD is proportional to $m_m r_m l_m$.

If the balancing radius r_m is known, balancing mass m_m can be obtained in magnitude and direction,

$$m_m = \frac{OD}{r_m l_m} \qquad OD = m_m r_m l_m$$

Thus m_m in plane M can be determined and angle θ_1 can be measured.

4. We can find other balancing mass m_l in plane L with the help of force polygon tabulated in column (4) of the table.



If the radius of 2nd balancing mass m_l is known m_l can be found in plane L and it's angle of [illegible]

Q2 A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5 cm, 6 cm, 7 cm and 6 cm respectively. The 2nd, 3rd and 4th masses revolve in planes 8 cm, 16 cm and 28 cm respectively measured from the plane of 1st mass, and are angularly located at 60°, 135°, 270° respectively measured anticlockwise from 1st mass. The shaft is dynamically balanced by two masses both located at 5 cm radii and revolving in 1st and 2nd masses and planes mid way both those of 3rd and 4th masses. Determine graphically the magnitude of the masses and their respective angular positions.

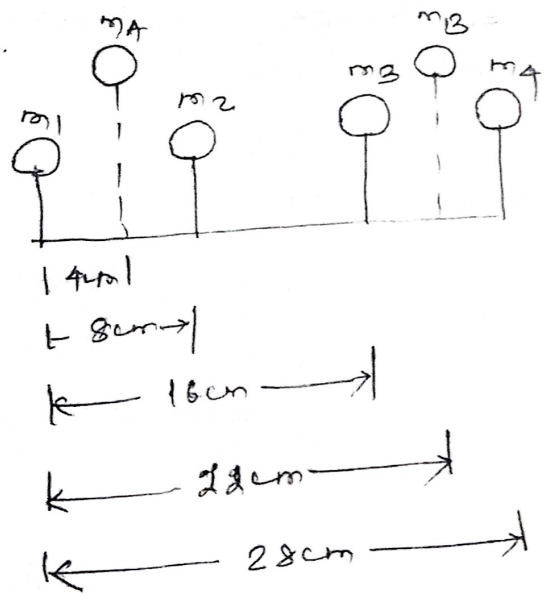
Given data :- $m_1 = 18$ kg $m_2 = 14$ kg $m_3 = 16$ kg

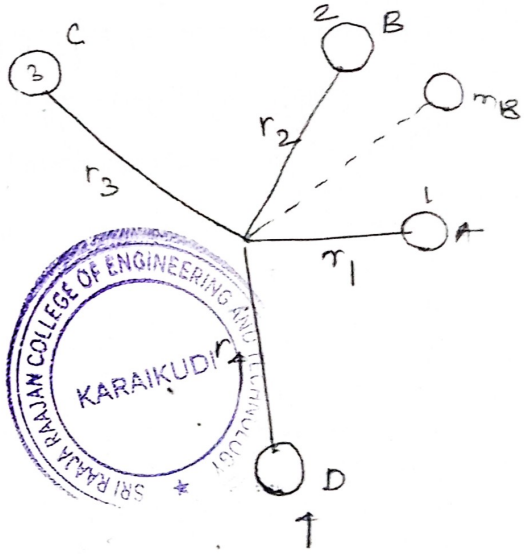
$m_4 = 12$ kg.

$r_1 = 5$ cm $r_2 = 6$ cm $r_3 = 7$ cm $r_4 = 6$ cm.

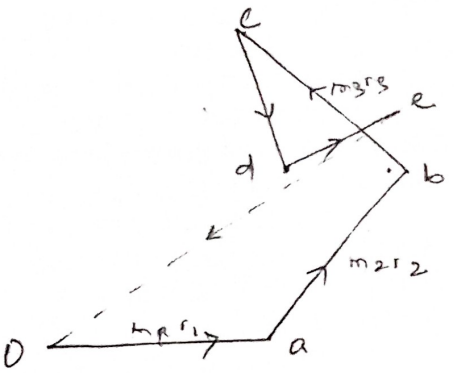
$\theta_1 = 0$ $\theta_2 = 60°$ $\theta_3 = 135°$ $\theta_4 = 270°$

Let the two balancing masses are m_A and m_B

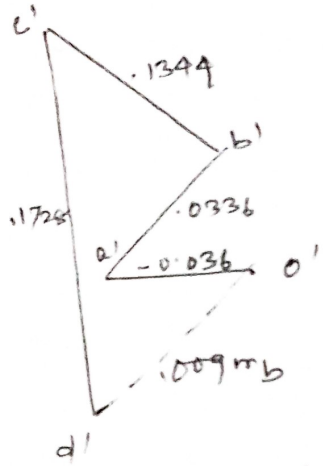




plane	mass (m) Kg	Radius (r) m	centrifugal force ÷ ω^2 (mr)	Distance from RP (l)	couple ÷ ω^2 (mrl)
1	18	0.05	0.9	-.04	-0.036
A	m_A	0.05	$0.05 m_A$	0	0
2	14	.06	0.84	.04	0.0336
3	16	.07	1.12	.12	0.1344
B	m_B	.05	$0.05 m_B$	.18	$.009 m_B$
4	12	.06	0.72	.24	0.1728



(force polygon)



$m_A = \frac{1.575}{0.05} = 31.5$ kg,

$\theta_A = 220°$ (from plane 1)

(2nd)

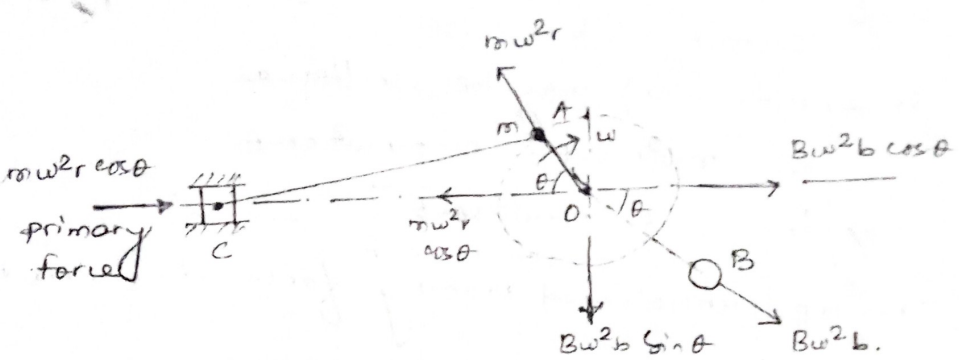
$m_B = \frac{o'd'}{.009}$

$= \frac{.12}{.009} = 13.3$

$\theta_B = 25°$

(1st)

Partial Primary Balancing :-



Consider a slider crank mechanism OAC. A primary unbalanced force $m\omega^2 r\cos\theta$ is required to accelerate the reciprocating mass, which acts along the direction from O to C. So balancing of primary force is considered equivalent to the component and parallel to the line of stroke, of the centrifugal force produced by an equal mass 'm' attached to the crank end and rotating at 'r' radius. To balance this force a rotating counter mass B is placed at a radius b, directly opposite to crank.

For complete balancing

$$B\omega^2 b\cos\theta = m\omega^2 r\cos\theta$$

$$\Rightarrow \boxed{B \cdot b = m \cdot r}$$

- However the vertical component of rotating mass B, of magnitude $B\omega^2 b\sin\theta$ remains unbalanced

Now the resultant disturbing force parallel to the line of stroke is

$$F_H = m\omega^2 r\cos\theta - B\omega^2 b\cos\theta$$

$$\Rightarrow \boxed{F_H = (m r - B \cdot b)\,\omega^2\cos\theta}$$

If $m \cdot r = B \cdot b$, the primary disturbing force is zero and the system will be unbalanced because of

Practically, a compromise is made and only a fraction c of reciprocating mass is balanced i.e,

$c \cdot m_r = B \cdot b$.

so equation (1) may be written as

$$F_H = m\omega^2 r\cos\theta - c \cdot m r\omega^2 \cos\theta$$

$$\Rightarrow \boxed{F_H = (1-c)\, m r\omega^2 \cos\theta} \quad \text{— (2)}$$

This is the unbalanced primary force acting along the line of stroke.

The unbalanced force ⊥r to the line of stroke is

$$F_V = B\omega^2 b \sin\theta = c m\omega^2 r \sin\theta \quad \text{— (3)}$$

so the resultant unbalanced force

$$F = \sqrt{F_H^2 + F_V^2}$$

$$\Rightarrow \boxed{F = m\omega^2 r \sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}} \quad \text{— (4)}$$

The value of c is kept betn 1/2 to 3/4.

The value of unbalanced force is min^m when

$c = \frac{1}{2}$

$$F_{min} = m\omega^2 r \sqrt{\left(\tfrac{1}{2}\right)^2\cos^2\theta + \left(\tfrac{1}{2}\right)^2\sin^2\theta}$$

$$\Rightarrow \boxed{F_{min} = \frac{m\omega^2 r}{2}}$$

Pb The following data relate to a single cylinder reciprocating engine:

mass of reciprocating parts = 40 kg.

mass of revolving parts = 30 kg at 180 mm radius.

speed = 150 rpm

stroke = 350 mm.

If 60% of reciprocating parts and all revolving parts are to be balanced, determine